Maths Calculations Policy 2022

Glenfield Primary School

PROGRESSION MAP
Addition

This must be viewed alongside the subtraction map so that connections can be made.

YR	Y1	Y2	Y3	Y4	Y5	Y6
Understanding the operation and related vocabulary						
understand addition as: -combining two quantities - increasing one quantity	understand addition as: - combining two or more quantities - increasing one quantity	continue to understand addition as: - combining two or more quantities - increasing one quantity	continue to develop understanding of addition			
		show that addition of two numbers can be done in any order (the commutative law)	understand the principles of the commutative and associative law	continue to understand the principles of the commutative and associative laws		**use their knowledge of the order of operations**
		recognise the inverse relationship between addition and subtraction	understand the inverse relationship between addition and subtraction	continue to understand the inverse relationship between addition and subtraction		
record using marks that they can interpret and explain	**read, write and interpret mathematical statements involving addition (+) and equals (=) signs** solve missing number problems	solve missing number problems.	solve missing number problems	continue to solve missing number problems	continue to solve missing number problems begin to use brackets	continue to solve missing number problems **explore the order of operations using brackets**
begin to use the vocabulary involved in adding *add, altogether, total, , ..more than ..*	understand the vocabulary related to addition *plus, the sum of*	understand the vocabulary related to addition	understand, read and spell vocabulary related to addition correctly *increase*	understand, read and spell vocabulary related to addition correctly	read, spell and pronounce mathematical vocabulary related to multiplication correctly	read, spell and pronounce mathematical vocabulary related to multiplication correctly
Recalling number facts						
recall addition facts to 5	recall and use addition facts to 10 fluently	**recall and use addition facts to 20 fluently, and derive and use related facts up to 100**	continue to recall and use addition facts to 20 fluently, and derive and use related facts beyond 100 80+50	continue to use knowledge of addition facts and place value to derive related facts 800+500	continue to use knowledge of addition facts and place value to derive related facts with numbers to one decimal place 1.2 + 0.7	continue to use knowledge of addition facts and place value to derive related facts with numbers to two decimal places

know number pairs with a total of 10 6 + ?	know number pairs with a total of 20	know complements to the next multiple of 10 52+? = 60 know pairs of multiples of 10 with a total of 100	know pairs of two-digit numbers with a total of 100	know complements to the next multiple of 100 568+? = 600	know complements to 1 0.83 + 0.17 = 1 recall pairs of three-digit numbers with a total of 1000	know complements to the next whole number 7.632 + ? = 8
Mental methods and mental methods with jottings						
find the total number of items in two groups by counting all of them add two single-digit numbers and count on to find the answer.	**add one-digit and two-digit numbers to 20, including zero** **represent and use number bonds within 20**	**add numbers using concrete objects, pictorial representations, and mentally, including:** * **a two-digit number and ones** * **a two-digit number and tens** * **two two-digit numbers** * **adding three one-digit numbers**	**add numbers mentally, including:** * **a three-digit number and ones** * **a three-digit number and tens** * **a three-digit number and hundreds**	continue to practise mental methods of addition with increasingly large numbers	**add numbers mentally with increasingly large numbers** add tenths, and one-digit whole numbers and tenths	**perform mental calculations, including with mixed operations, large numbers and decimals** add positive and negative integers (in contexts such as temperature)
Formal written layout						
		add numbers with up to two digits, using number lines and partitioning	**add numbers with up to three digits, using formal written methods of columnar addition**	**add numbers with up to 4 digits using the formal written method of columnar addition where appropriate**	**add whole numbers with more than 4 digits, including using formal written methods**	practise addition for larger numbers, using formal written methods
				add decimals to 2 decimal places (in the context of money or measures)	add decimals, including a mix of whole numbers and decimals and decimals with different numbers of decimal places	continue to practice addition calculations with decimals (up to 3 decimal places)
Estimating and checking						
		check calculations by adding in a different order	**estimate the answer to a calculation** **use inverse operations to check answers** use equivalent calculations to check answers	**estimate the answer to a calculation** **use inverse operations to check answers** use equivalent calculations to check answers	**use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy** continue to use appropriate strategies to check answers	**use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.** continue to use appropriate strategies to check answers

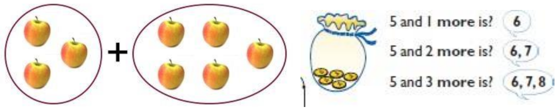
ADDITION: Y1

Understanding the operation and related vocabulary.

Understanding the operation

Understand addition as:

- Combining two or more quantities.
- Increasing one quantity.



Read, write and interpret mathematical statements involving addition (+) and equals (=) sign.

14+5=19   17=9+8

Solve missing number problems

11+☐=18   ☐=13+2   13=☐+☐

Understand addition and subtraction as related operations. E.g. 7 + 3 = 10 is related to 10 – 3 = 7

When introduced to the equals sign, children should see it as signifying equality. They should become used to seeing it in different positions.

Mental Calculations

Number facts

Recall and use addition facts to 10 fluently

the total of 6 and 3   6 plus 2   4 more than 5

Know number pairs with a total of 20

16+☐ = 20   20 = 3+☐

Mental methods and jottings

Add one-digit and two-digit numbers to 20, including zero using concrete objects, pictorial representation and mentally.

Represent and use number bonds within 20, experiencing the = sign in different positions.

Counting on (sequencing)

12 + 3 (by counting on in ones; 13, 14, 15)

With Jottings:

Progress to crossing the tens boundary

18 + 5 (by partitioning 5 to bridge the tens boundary; + 2, + 3)

Partitioning

5 + 7 (by partitioning 7 in to 5 and 2)

5 + 5 + 2

Use bundles of straws and Dienes to model partitioning teen numbers into tens and ones and develop their understanding of place value.

Children have opportunities to explore partitioning numbers in different ways.

e.g. 7 = 6 + 1, 7 = 5 + 2, 7 = 4 + 3

Written Calculations

No formal written layout. Children record their maths using pictorial representations, number lines and mathematical statements.

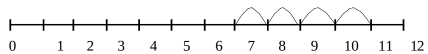
Counting and Combining sets of Objects

5+7=12

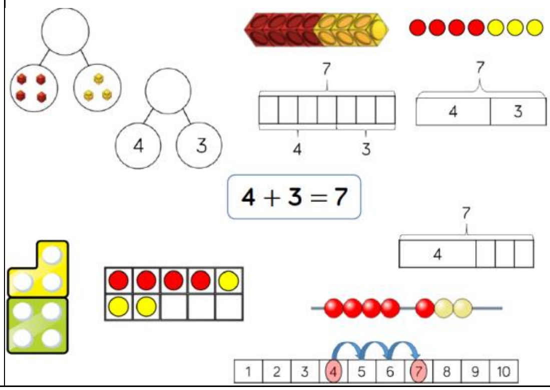
Add one-digit and two-digit numbers to 20, including zero

7 + 4

OR



OR



Vocabulary

Understand the vocabulary related to addition:

Addition, add (+), forwards, put together, more than, total, altogether, distance between, difference between, equals = same as, most, pattern, odd, even, digit, counting on, plus, the sum of

Generalisations

- True or false? Addition makes numbers bigger.
- True or false? You can add numbers in any order and still get the same answer.

Some Key Questions

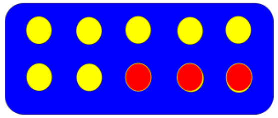
How many altogether? How many more to make…? I add …more. What is the total? How many more is… than…? How much more is…? One more, two more, ten more…

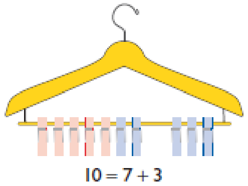
What can you see here?

Is this true or false?

What is the same? What is different?

Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10.



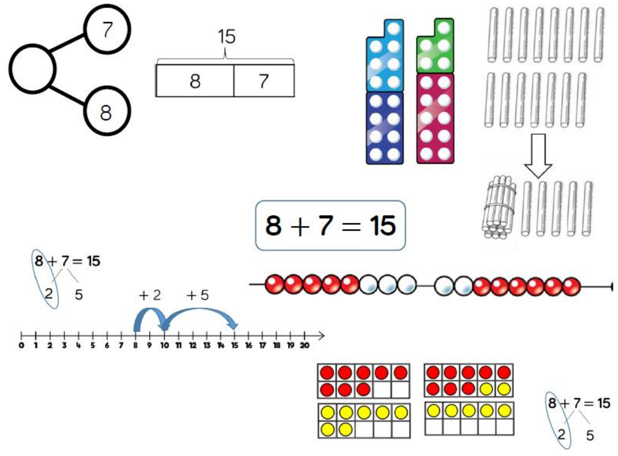


Using known facts and place value

15 + 4

5 + 4 = 9 so 15 + 4 = 19

OR (also applicable in year 2)



ADDITION: Y2

Understanding the operation and related vocabulary.

Understanding the operation

Continue to understand addition as:

- Combining two or more quantities.
- Increasing one quantity.

Show that addition of two numbers can be done in any order (commutative law)

Recognise that 5 + 27 is equal to 27 + 5

Continue to recognise the inverse relationship between and addition and subtraction using numbers up to 20.

Write the related number sentences

15+2=17 2+15=17 17=15+2 17=2+15

17-2=15 17-15=2 2=17-15 15=17-2

Solve missing number problems

17+☐=27 ☐=21+4 10=☐+☐

Vocabulary

Understand the vocabulary related to addition

+, add, addition, more, plus, make, sum, total, altogether, how many more to make…? how many more is… than…? how much more is…? =, equals, sign, is the same as, Tens, ones, partition

Near multiple of 10, tens boundary, More than, one more, two more… ten more… one hundred more

Generalisation

- Noticing what happens when you count in tens (the digits in the ones column stay the same)
- Odd + odd = even; odd + even = odd; etc
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot

Mental Calculations

Number facts

Recall and use number facts to 20 fluently and derive and use related facts up to 100.

7 add 8 4 more than 9 50 plus 30 the sum of 40 and 50

Know complements to the next multiple of 10.

52+☐ = 60 76+☐ = 80

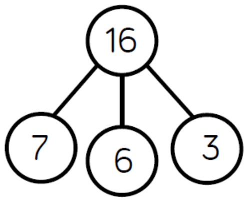
Know pairs of multiples of 10 with a total of 100.

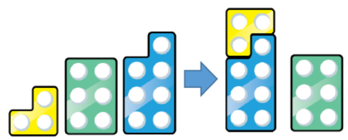
60+☐ = 100 100 = 70+☐

Mental methods and jottings

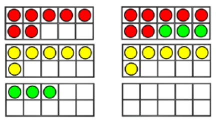
Add numbers using concrete objects, pictorial representations, and mentally, including:

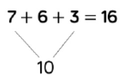
* a two-digit number and ones
* a two-digit number and tens
* two two-digit numbers
* adding three one-digit numbers

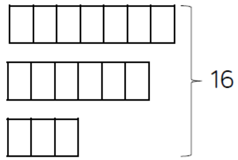




7 + 6 + 3 = 16





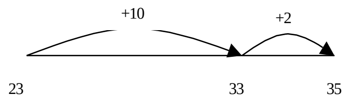


Written Calculations

Continue to use number lines to develop understanding of:

Counting on in tens and ones

23 + 12 = 23 + 10 + 2
= 33 + 2
= 35

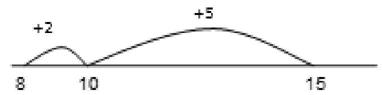


Partitioning and bridging through 10.

The steps in addition often bridge through a multiple of 10

e.g. Children should be able to partition the 7 to relate adding the 2 and then the 5.

8 + 7 = 15



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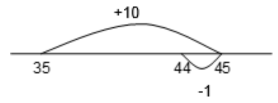
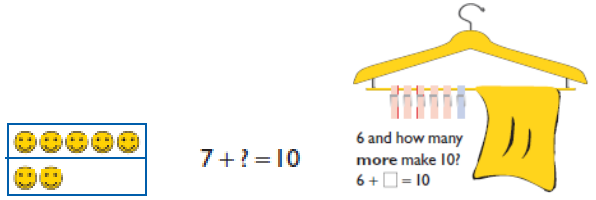
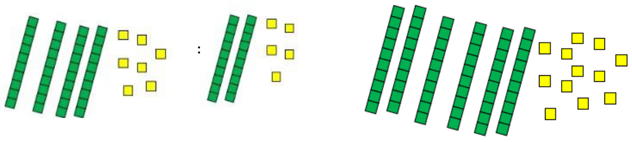
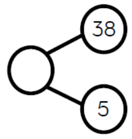
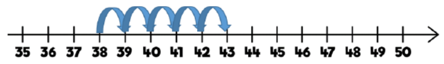
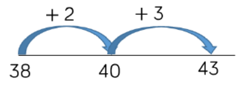
	Counting on 37 + 20 (by counting on in tens; 47, 57) With Jottings Begin by not crossing the tens boundary 42 + 23 (by partitioning the second number and counting on; + 20, + 3) Progress to crossing the tens boundary 47 + 15 (by partitioning the second number and counting on; + 10, +3, +2) **Partitioning** 23 + 12 (20 + 10 = 30, 3 + 2 = 5 then 30 + 5 = 35) With Jottings Begin by not crossing the tens boundary 42 + 23 (40 + 20 = 60; 3 + 2 = 5 then 60 + 5)	**Adding 9 or 11 by adding 10 and adjusting by 1** e.g. Add 9 by adding 10 and adjusting by 1 35 + 9 = 44 

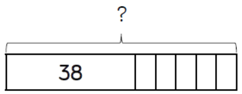
• Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems. This understanding could be supported by images such as this.  **Some Key Questions** How many altogether? How many more to make…? How many more is… than…? How much more is…? Is this true or false? If I know that 17 + 2 = 19, what else do I know? (e.g. 2 + 17 = 19; 19 – 17 = 2; 19 – 2 = 17; 190 – 20 = 170 etc). What do you notice? What patterns can you see?	Progress to crossing the tens boundary 47 + 15 (40 + 10 = 50, 7 + 5 =12 then 50 +12 = 62) **Adjusting** 34 + 9 (adding 10 then subtracting 1) With Jottings 45 + 19 (by adding 20 and subtracting 1) **Using known facts and place value:** 63 +4 3+4=7 so 63+4=67 **Estimating:** Check calculations by adding in a different order check 27 + 15 (27 + 10 + 5) with 15 + 20 + 7	Partitioning through use of physical resources **47 + 25 = 72**  Adding two-digit numbers by partitioning each number and recombining 25 + 32 20 + 30 = 50 (tens) 5 + 2 = 7 (ones) 50 + 7 = 57 Adding two-digit numbers by partitioning the second number 25 + 32 25 + 30 = 55 55 + 2 = 57

Adding 1-digit and 2-digit numbers to 100 (also applicable in year 3)

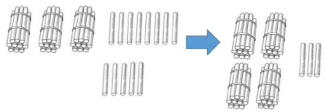






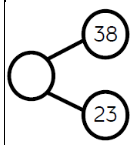


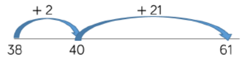
$38 + 5 = 43$

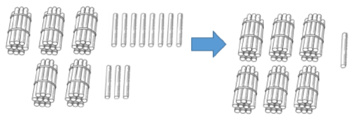


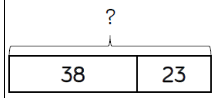
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Add two 2-digit numbers to 100 (also applicable in year 3)

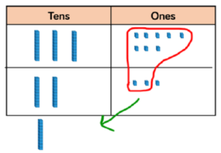




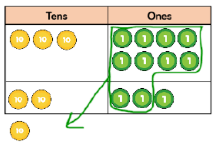




$38 + 23 = 61$



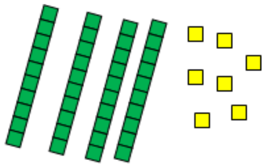
$$\begin{array}{r} 38 \\ +\ 23 \\ \hline 61 \\ \hline {\scriptstyle 1} \end{array}$$

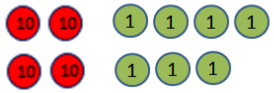


ADDITION: Y3		
Understanding the operation and related vocabulary.	**Mental Calculations**	**Written Calculations**
Understanding the operation Understand the principles of the commutative and associative law: Recognise that 45 + 36 is equal to 36 + 45 Recognise that if calculating 13 + 14 + 9 the numbers can be combined in any order Understand the inverse relationship between addition and subtraction 45+22=67 22+45=67 67=45+22 67=22+45 67-22=45 67-45=22 22=67-45 45=67-22 Solve missing number problems 62+☐=74 ☐=45+32 ☐+☐=50 100 - 3 = 67 + ☐ 45 < ☐ + 6 ☐+☐ > 54 + 9 **Vocabulary** Hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange See also Y1 and Y2 **Generalisations** Noticing what happens to the digits when you count in tens and hundreds. Odd + odd = even etc (see Year 2) Inverses and related facts – develop fluency in finding related addition and subtraction facts. Develop the knowledge that the inverse relationship can be used as a checking method. **Key Questions** What do you notice? What patterns can you see? When comparing two methods alongside each other: What's the same? What's different? Look at this number	**Number facts** Continue to recall and use addition facts to 20 fluently, and derive and use related facts beyond 100 7 add 9, 80 plus 70, the sum of 90 and 60, 30 more than 110 Know pairs of two-digit numbers with a total of 100 74 +☐ = 100 100 = 59 +☐ **Mental methods and jottings** Add numbers mentally, including: * a three-digit number and ones * a three-digit number and tens * a three-digit number and hundreds **Counting On (Sequencing)** 137 + 50 (by counting on in tens; 147, 157, 167, 177, 187 With jottings: 345 + 37 (by partitioning the second number and counting on; +30, +5, +2) **Partitioning:** 236 + 33 (30+30=60, 6+3=9, 200+60+9=269) With jottings: 236 + 85 (80+30=110, 6+5=11, 200+110+11=321) **Adjusting:** 234 + 99 (by adding 100 and subtracting 1) With jottings: 334 + 59 (by adding 60 and subtracting 1) **Using known facts and place value:** 282 + 7 2+7=9 so 282+7= 289	For those that need reinforcement, begin with using a number line and partitioning the second number. **Partitioning** Partition both numbers and recombine. Count on by partitioning the first number only: 247 + 125 = 200 + 100 = 300 40 + 20 = 60 7 + 5 = 12 300 + 60 + 12 = 372 Move on to partitioning the second number only: 247 + 125 = 247 + 100 = 347 347 + 20 = 367 367 + 5 = 372 Children need to be secure adding multiples of 100 and 10 to any three-digit number including those that are not multiples of 10. **Towards a Written Method** Introduce expanded column addition modelled with place value counters or Dienes. 247 + 12 200 40 7 10 2 ――――― 200 50 9 ―――――

in the formal method; can you see where it is in the expanded method / on the number line?

Manipulatives can be used to support mental imagery and conceptual understanding. Children need to be shown how these images are related eg. What's the same? What's different?





<u>Estimating:</u>

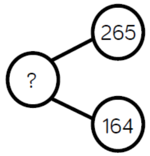
Estimate the answer to a calculation

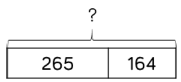
139 + 58 is approximately 150 + 50

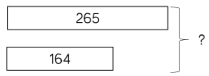
Use inverse operations or equivalent calculations to check answers

236 + 85 by adding in a different order e.g. 200 + 85 + 36

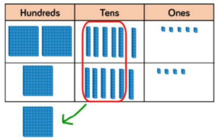
<u>Add numbers with up to 3 digits</u>



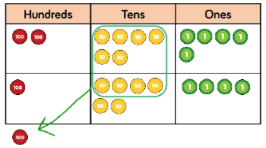


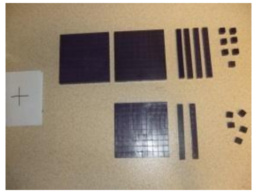


$265 + 164 = 429$



$$\begin{array}{r} 265 \\ +\,164 \\ \hline 429 \\ {\scriptstyle 1} \end{array}$$





<u>Add numbers with up to three digits, using formal written methods of columnar addition for higher attainers</u>

ADDITION: Y4

Understanding the operation and related vocabulary.

Understanding the operation

Continue to understand the principles of the commutative and associative laws

Recognise that 342 + 187 is equal to 187 + 342

Recognise that if calculating 46 + 39 + 14 the numbers can be combined in any order

Continue to understand the inverse relationship between addition and subtraction

256+92=348 92+256=348 348=256+92

348=92+256 348-256=92 348-92=256

92= 348-256 256=348-92

Continue to solve missing number problems

456+□=673 □=300+176 □+□=125

1000 - 103 = 450 + □ 450 < □ + 60 □+□ > 345+199

Vocabulary

add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make..? how much more? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? Equals sign, is the same as.

Generalisations

Investigate when re-ordering works as a strategy for subtraction. Eg. 20 – 3 – 10 = 20 – 10 – 3, but 3 – 20 – 10 would give a different answer.

Some Key Questions

What do you notice?

What's the same? What's different?

Can you convince me?

Mental Calculations

Number facts

Continue to use knowledge of addition vocabulary

5000 add 3000, 700 plus 800, the sum of 700 and 600, 300 more than 1200

Know complements to the next multiple of 100

568+□ = 600 749+□ = 800

Continue to practise mental methods of addition with increasingly large numbers.

Mental methods and jottings

Counting On (Sequencing):

534 + 150 (by partitioning the second number and counting on; +100, +50)

With jottings:

675+28 (by partitioning the second number and counting on; +25, +3)

Partitioning:

87 + 46 (80+40=120, 7+6=13, 120+13=133)

With jottings:

456 + 362 (400+300=700, 50+60=110, 6+2=8, 700+110+8=818)

Adjusting:

1435 + 199 (by adding 200 and subtracting 1)

With jottings:

1764+79 (by adding 80 (+40, +40) and subtracting 1)

Using known facts and place value:

6060 + 47

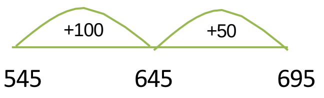
60+47= 107 so 6060+47=6107

Written Calculations

Written methods (progressing to 4-digits)

For those that need reinforcement, begin with using a number line and partitioning the second number.

545 + 150



Expanded column addition modelled with place value counters where appropriate

247 + 125 = 372

	10			
200	40	7		
100	20	5		
300	70	2	=	372

Progress to calculations with 4-digit numbers using the formal compact written method of columnar addition where appropriate.

Compact written method

Extend to numbers with at least four digits. When carrying part of answer, carry it over under the calculation as shown in the example.

How do you know?

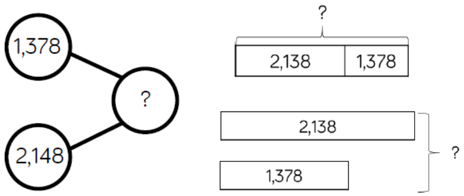
<u>Estimating:</u>

Estimate the answer to a calculation

2467 + 1729 is approximately 2500 + 1500

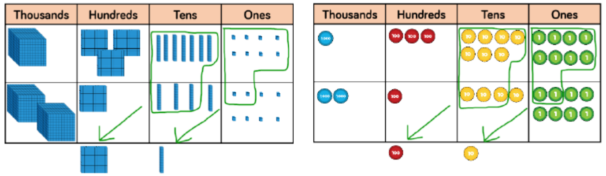
Use inverse operation or an equivalent calculations to check answers

<u>Add numbers with up to 4 digits</u>



$$\begin{array}{r} 1\,3\,7\,8 \\ +\,2\,1\,4\,8 \\ \hline 3\,5\,2\,6 \\ {\scriptstyle 1\;1\;\;} \end{array}$$

1,378 + 2,148 = 3,526



$$\begin{array}{r} 4517 \\ +\;2634 \\ {\scriptstyle 1\;\;1\;\;} \\ \hline 7151 \\ \hline \end{array}$$

Children should be able to make the choice of reverting to expanded methods if experiencing any difficulty. Extend to up to two places of decimals (same number of decimals places) and adding several numbers (with different numbers of digits).

$$\begin{array}{r} 72.8 \\ +\;54.6 \\ {\scriptstyle 1\;\;1\;\;\;\;} \\ \hline 127.4 \\ \hline \end{array}$$

ADDITION: Y5

<table>
<tr><th>Understanding the operation and related vocabulary.</th><th>Mental Calculations</th><th>Written Calculations</th></tr>
<tr><td>

Understanding the operation
Continue to solve missing number problems
6.5+☐=10.7 ☐=8.4+3.7 ☐+☐=4.2
7.3+2.9 = 9.9 + ☐ 5.2 < ☐ - 0.9 ☐ - ☐ > 7.2-1.9

Begin to use brackets
(10+3) x 7 = ☐ ☐=10 + (0.4 x 8)

Vocabulary
tens of thousands boundary,
Also see previous years

Generalisation
Sometimes, always or never true? The difference between a number and its reverse will be a multiple of 9.
What do you notice about the differences between consecutive square numbers?
Investigate a – b = (a-1) – (b-1) represented visually.

Some Key Questions
What do you notice?
What's the same? What's different?
Explain why digits are carried over to the next columns.

</td><td>

Number facts
Continue to use knowledge of addition facts and place value to derive related facts with numbers to one decimal place
1.2 plus 0.7, the total of 0.8 and 0.9, the sum of 0.2 and 1.3, 0.3 more than 1.7

Know complements to 1
0.78 + ☐ = 1 0.52 + ☐ = 1

Recall pairs of three-digit numbers with a total of 1000
456 + ☐ = 1000 1000 = ☐ + 825

Mental methods and jottings
Add numbers mentally with increasingly large numbers.
Add tenths, and one-digit whole numbers and tenths.

Counting on (sequencing):
4.3 + 1.5 (by partitioning the second number and counting on; +1, +0.5)
With jottings:
19.7 + 2.6 (by partitioning the second number and counting on; +2, +0.3, +0.3)

Partitioning:
3.6 + 1.7 (3+1=4, 0.6+0.7=1.3, 4+1.3=5.3)
With jottings:
18.7 + 14.8 (18+14=32, 0.7+0.8=1.5, 32+1.5=33.5)

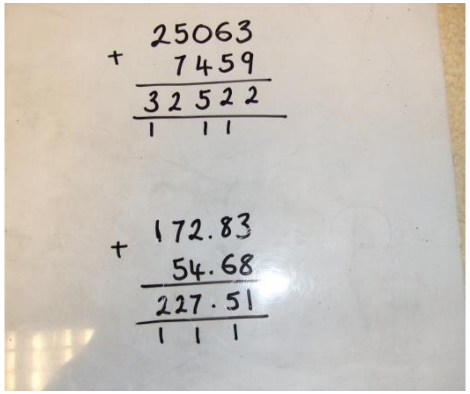
Adjusting:
8.3 + 1.9 (by adding 2 and subtracting 0.1)
With jottings:
14.6 + 3.9 (by adding 4 and subtracting 0.1)

</td><td>

Add whole numbers with 5 digit numbers, including using formal written methods.

Written methods (progressing to more than 4-digits)
As year 4, progressing when understanding of the expanded method is secure, children will move on to the formal columnar method for whole numbers and decimal numbers as an efficient written algorithm.

Leave a gap under the calculation for digits being carried over (see year 4). Move on to carrying underneath the whole calculation by the end of year 5.



Place value counters can be used alongside the columnar method to develop understanding of addition with decimal numbers.

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</table>

Using known facts and place value:

7.5 + 2.6

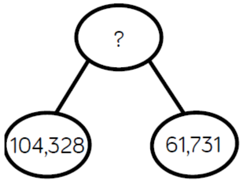
7.5 + 2.5 = 10 so 7.5 + 2.6 = 10.1

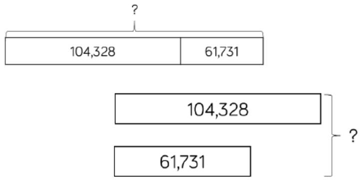
Estimating

Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy

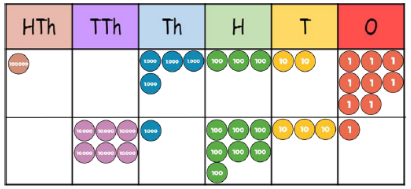
25 063 + 7459 is approximately 25 000 + 7500

Add numbers with more than 4 digits (also applicable in year 6)



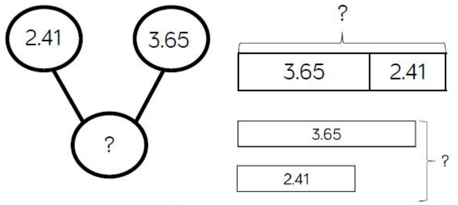


104,328 + 61,731 = 166,059



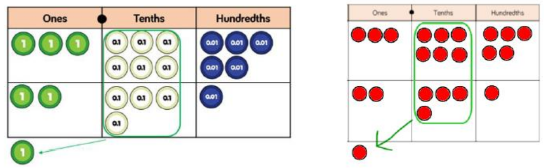
1	0	4	3	2	8
+	6	1	7	3	1
1	6	6	0	5	9
		1			

Add with up to 3 decimal places



$$\begin{array}{r} 3.65\\ +\,2.41\\ \hline 6.06\\ \hline {}_{1} \end{array}$$

3.65 + 2.41 = 6.06



Continue to use appropriate strategies to check answers
check 8.3 + 1.9 by adding in a different order
8.3 + 2 – 0.1 or 8.3 + 0.7 + 1.2

ADDITION: Y6		
Understanding the operation and related vocabulary.	**Mental Calculations**	**Written Calculations**
<u>Understanding the operation</u> Use their knowledge of the order of operations. Understand that when there are no brackets in an expression, do multiplication or division before addition or subtraction. Understand that if the operations are at the same level of priority, work out the example from left to right. Continue to solve missing number problems 0.63+☐=0.85 ☐=0.5+0.33 ☐+☐=0.71 0.89 + 0.3 = 0.6 + ☐ 0.75 < ☐ + 0.06 ☐ + ☐ > 0.74 + 0.07 Explore the order of operations using brackets compare 14 – (3 + 5) with (14 – 3) + 5 **<u>Vocabulary</u>** See previous years Read, spell and pronounce mathematical vocabulary related to addition correctly **<u>Generalisations</u>** Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as PEMDAS, or could be encouraged to design their own ways of remembering. Sometimes, always or never true? Subtracting numbers makes them smaller.	**<u>Number facts</u>** Continue to use knowledge of addition facts and place value to derive related facts with numbers to two decimal places 0.09 plus 0.04, the total of 0.09 and 0.08, the sum of 0.06 and 0.12 , 0.04 more than 1.13 Know complements to the next whole number 4.83 + ☐ = 5 7.125 + ☐ = 8 **<u>Mental methods and jottings</u>** Perform mental calculations, including with mixed operations, large numbers and decimals Add positive and negative integers (in contexts such as temperature) a 6°C temperature rise from -4°C **<u>Counting On (Sequencing):</u>** 6.46 + 2.03 (by partitioning the second number and counting on; +2, +0.03) With jottings: 18.7 + 5.64 (by partitioning the second number and counting on; +5, +0.3, +0.34) **<u>Partitioning:</u>** 3.4 + 2.77 (3+2=5, 0.4+0.7=1.1, 5+1.1+0.07=6.17) With jottings: 27.34 + 5.78 (27+5=33, 0.3+0.7=1, 0.04+0.08=0.12, 33+1+0.12=34.12) **<u>Adjusting:</u>** 6.73 + 0.99 (by adding 1 and subtracting 0.01) With jottings:	**<u>Written methods</u>** As year 5, progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with columnar method to be secured. 657 982 + 54 976 Continue calculating with decimals, including those with different numbers of decimal places 73.82 + 17.382 **<u>Problem Solving</u>** Teachers should ensure that pupils have the opportunity to apply their knowledge in a variety of contexts and problems (exploring cross curricular links) to deepen their understanding

<u>Some Key Questions</u> What do you notice? What's the same? What's different? Can you convince me? How do you know?	17.4 + 5.09 (by adding 5.1 and subtracting 0.01) **<u>Using Known Facts And Place Value:</u>** 0.64 + 0.36 64 + 36 = 100 so 0.64 + 0.36 = 1 **<u>Estimating:</u>** Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy. 73.82 + 17.382 is approximately 74 + 17 Continue to use appropriate strategies to check answers check 3.4 + 2.77 by adding in a different order partition or add 3 and adjust	

PROGRESSION MAP
Subtraction

This must be viewed alongside the addition map so that connections can be made.

YR	Y1	Y2	Y3	Y4	Y5	Y6
Understanding the operation and related vocabulary						
understand subtraction as: - **'taking away'** - removing part of a set & reduction - **'difference'** – comparison & how much more is needed	understand subtraction as: - **'taking away'** - removing part of a set & reduction - **'difference'** – comparison & how much more is needed	understand subtraction as: - **'taking away'** - removing part of a set & reduction - **'difference'** – comparison & how much more is needed - complement of a set	continue to develop understanding of subtraction			
		show that subtraction of one number from another cannot be done in any order	understand that the principles of the commutative and associative laws do not apply to subtraction	continue to understand that the principles of the commutative and associative laws do not apply to subtraction		**use their knowledge of the order of operations**
		recognise the inverse relationship between addition and subtraction	understand the inverse relationship between addition and subtraction	continue to understand the inverse relationship between addition and subtraction		
record using marks that they can interpret and explain	**read, write and interpret mathematical statements involving subtraction (-) and equals (=) signs;** solve missing number problems	solve missing number problems	solve missing number problems	continue to solve missing number problems	continue to solve missing number problems begin to use brackets	continue to solve missing number problems **explore the order of operations using brackets**
begin to use the vocabulary involved in subtracting *take away, subtract, how many are left, how many more to make, how many more, how many fewer, …less than .., leave, how many have gone*	understand the vocabulary related to subtraction *minus, the difference between, how much more is … than …, how much less is .. than …*	understand the vocabulary related to subtraction	understand, read and spell vocabulary related to subtraction correctly *decrease*	understand, read and spell vocabulary related to subtraction correctly	read, spell and pronounce mathematical vocabulary related to subtraction correctly	read, spell and pronounce mathematical vocabulary related to subtraction correctly
Recalling number facts						
recall subtraction facts to 5	recall and use subtraction facts to 10 fluently	**recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100**	continue to recall and use subtraction facts to 20 fluently, and derive and use related facts beyond 100	continue to use knowledge of subtraction facts and place value to derive related facts	continue to use knowledge of subtraction facts and place value to derive related facts with numbers to one decimal place	continue to use knowledge of subtraction facts and place value to derive related facts with

						numbers to two decimal places
know number pairs with a total of 10 and derive related subtraction facts	know number pairs with a total of 20 and derive related subtraction facts	know complements to the next multiple of 10 know pairs of multiples of 10 with a total of 100 and derive related subtraction facts	know pairs of two-digit numbers with a total of 100 and derive related subtraction facts	know complements to the next multiple of 100	know complements to 1 recall pairs of three-digit numbers with a total of 1000 and derive related subtraction facts	know complements to the next whole number
Mental methods and mental methods with jottings						
find how many are left when some are taken away subtract two single-digit numbers and count back to find the answer. partition a given number of objects (up to 10) into 2 groups	**subtract one-digit and two-digit numbers to 20, including zero** **represent and use number bonds within 20** partition a given number of objects (up to 20) into 2 groups	**subtract numbers using concrete objects, pictorial representations, and mentally, including:** * **a two-digit number and ones** * **a two-digit number and tens** * **two two-digit numbers**	**subtract numbers mentally, including:** * **a three-digit number and ones** * **a three-digit number and tens** * **a three-digit number and hundreds**	continue to practise mental methods of subtraction with increasingly large numbers	**subtract numbers mentally with increasingly large numbers** subtract tenths, and one-digit whole numbers and tenths	**perform mental calculations, including with mixed operations, large numbers and decimals** calculate intervals across zero
Formal written layout						
		Subtract numbers with up to two digits, using partitioning and number lines.	**subtract numbers with up to three digits, using formal written methods of columnar subtraction**	**subtract numbers with up to 4 digits using the formal written method of columnar subtraction where appropriatio**	**subtract whole numbers with more than 4 digits, including using formal written methods**	practise subtraction for larger numbers, using formal written methods
				subtract decimals to 2 decimal places (in the context of money or measures)	subtract decimals, including a mix of whole numbers and decimals and decimals with different numbers of decimal places	continue to practice subtraction calculations with decimals (up to 3 decimal places)
Estimating and checking						
		Use inverse operations to check answers	**estimate the answer to a calculation** **use inverse operations to check answers** use equivalent calculations to check answers	**estimate the answer to a calculation** **use inverse operations to check answers** use equivalent calculations to check answers	**use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy** continue to use appropriate strategies to check answers	**Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.** continue to use appropriate strategies to check answers

SUBTRACTION: Y1

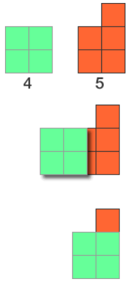
Understanding the operation and related vocabulary.

Understanding the operation

understand subtraction as:

'taking away' - removing part of a set & reduction

'difference' – comparison & how much more is needed



Read, write and interpret mathematical statements involving subtraction (-) and equals (=) signs ;

14-3=11 9=16-7

To begin with, children are given number sums, before moving onto writing their own sums.

Solve missing number problems e.g.

11-☐=8 ☐=13-2 3=☐-☐

Vocabulary

Subtraction, subtract, take away, minus, distance between, difference between, more than, minus, less than, equals = same as, most, least, pattern, odd, even, digit,

Generalisations

- True or false? Subtraction makes numbers smaller

Mental Calculations

Number facts

Recall and use subtraction facts to 10 fluently e.g.

6 minus 3 8 subtract 2 4 less than 9

Know number pairs with a total of 20 and derive related subtraction facts e.g.

20+0, 20-1, 20-2, 20-3 …

Mental methods and jottings

Subtract one-digit and two-digit numbers to 20, including zero

Represent and use number bonds within 20

Partition a given number of objects (up to 20) into 2 groups e.g.

Partition 15 into 7 and 8, 9 and 6 ….

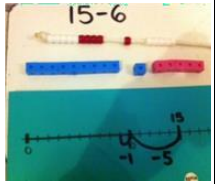
Counting back

15-3 (by counting back 3 in ones; 14, 13, 12)

With jottings

15 – 6 (by counting back in ones or partitioning 6 to bridge the tens boundary; -5, -1)

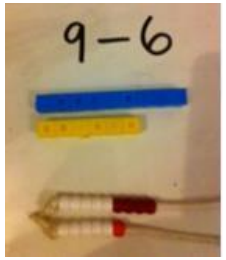
Progress to crossing the tens boundary

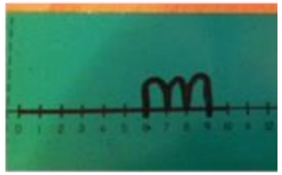


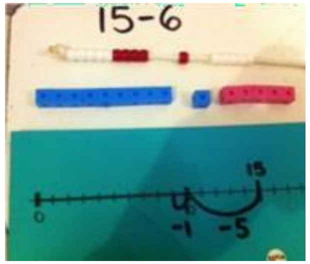
Written Calculations

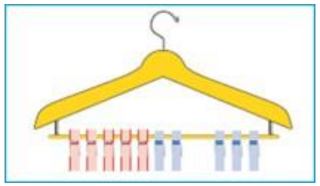
No formal written layout.

Children will be recording their mathematics using pictorial representations, number lines and mathematical statements.





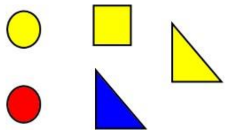




- When introduced to the equals sign, children should see it as signifying equality. They should become used to seeing it in different positions.

Children could see the image below and consider, “What can you see here?” e.g.

3 yellow, 1 red, 1 blue. 3 + 1 + 1 = 5
2 circles, 2 triangles, 1 square. 2 + 2 + 1 = 5
I see 2 shapes with curved lines and 3 with straight lines. 5 = 2 + 3
5 = 3 + 1 + 1 = 2 + 2 + 1 = 2 + 3



Some Key Questions

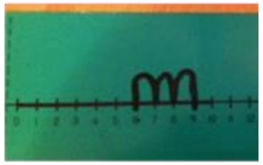
How many more to make…? How many more is… than…? How much more is…? How many are left/left over? How many have gone? One less, two less, ten less… How many fewer is… than…? How much less is…?
What can you see here?
Is this true or false?

Counting up

9 – 6 (by counting up from 6 to 9 in ones; 7, 8, 9)

With jottings

19 – 14 (by counting up from 14 to 19 in ones; 15, 16, 17, 18, 19)



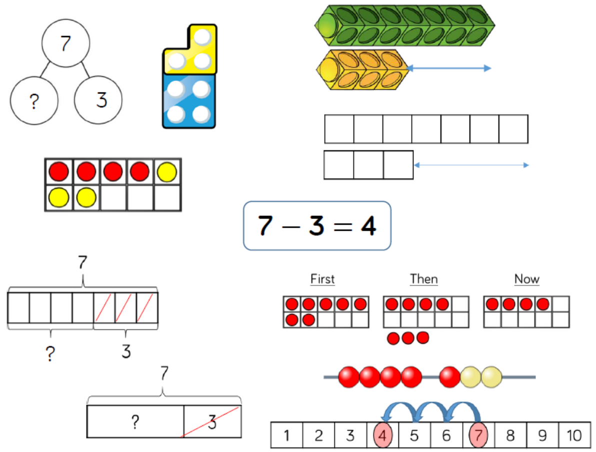
Using known facts and place value

6 – 4 = 2 so 16-4 = 12

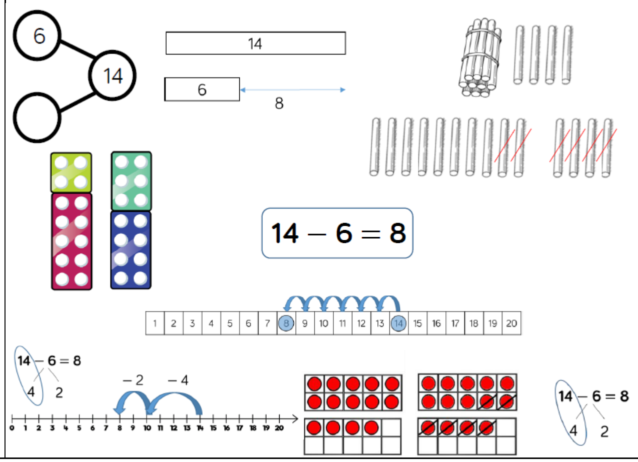
Reinforcement of number facts, bonds, etc to be reinforced through the use of number songs and other ‘active’ exercises.

Use of concrete and pictorial representation

Use of resources to support children’s mental subtraction and to help establish what subtraction physically is/looks like e.g. cubes, straws, counters, money, number squares, jottings/pictures.



OR (also applicable in year 2)



SUBTRACTION: Y2

Understanding the operation and related vocabulary.

Understanding the operation

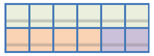
Understand subtraction as:

- taking away
- comparison (finding the difference)
- partitioning a set

Show that subtraction of one number from another cannot be done in any order

Recognise that 5-3 is different from 3-5

Recognise the inverse relationship between addition and subtraction



Write the related number sentences

5+2=7 2+5=7 7=5+2 7=2+5
7-2=5 7-5=2 2=7-5 5=7-2

Solve missing number problems e.g.

27-☐=17 ☐=21-4 10=☐-☐

Vocabulary

Subtraction, subtract, take away, difference, difference between, minus, Tens, ones, partition, near multiple of 10, tens boundary, less than, one less, two less... ten less... one hundred less, more, one more, two more... ten more... one hundred more

Generalisation

Mental Calculations

Number facts

Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100 e.g.

15 subtract 8 4 less than 12 80 minus 30 90 take 50

Know complements to the next multiple of 10 e.g.

52+☐ = 60 52+☐ = 80

Know pairs of multiples of 10 with a total of 100 and derive related subtraction facts e.g.

100-10, 100-20, 100-30 ...

Mental methods and jottings

Subtract numbers using concrete objects, pictorial representations, and mentally, including:

* a two-digit number and ones
* a two-digit number and tens
* two two-digit numbers

Counting back

57 – 20 (by counting back in tens; 47, 37)

With jottings

57 – 23 (by partitioning the second number and counting back; -20, -3)

Begin by not crossing the tens boundary

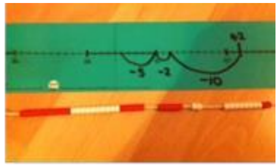
42 – 17 (by partitioning the second number and counting back; -10, -2, -5)

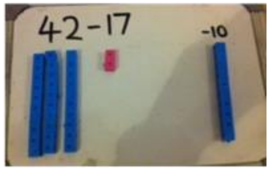
Progress to crossing the tens boundary

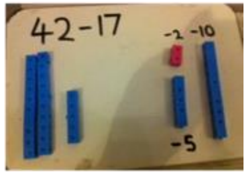
See image on next page

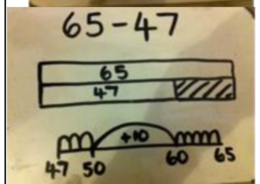
Written Calculations

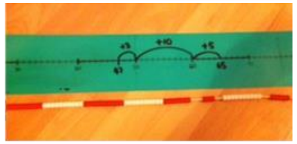
Children will be recording their mathematics using pictorial representations, number lines and mathematical statements.









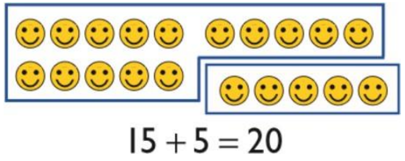


Counting back through partitioning

Use of partitioning to subtract 2-digit numbers in portions i.e. taking away tens, and then ones e.g.

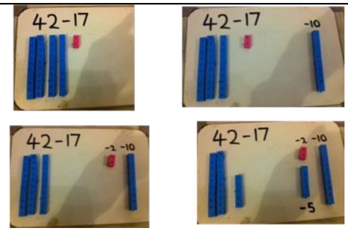
54 – 23
54 – 20 = 34

- Noticing what happens when you count back in tens (the digits in the ones column stay the same)
- Odd – odd = even; odd – even = odd; etc
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- Recognise and use the [inverse](#) relationship between addition and subtraction and use this to check calculations and missing number problems. This understanding could be supported by images such as this.



Some Key Questions

How many more to make…? How many more is… than…? How much more is…? How many are left/left over? How many fewer is… than…? How much less is…?
Is this true or false?
If I know that 7 + 2 = 9, what else do I know? (e.g. 2 + 7 = 9; 9 – 7 = 2; 9 – 2 = 7; 90 – 20 = 70 etc).
What do you notice? What patterns can you see?

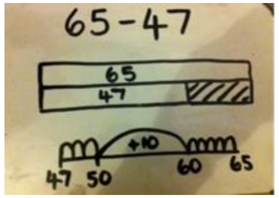


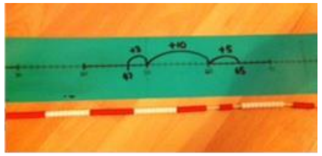
Counting up

31 - 28 (by counting up from 28 by bridging the tens boundary; +2, +1)

With jottings

65 – 47 (by counting up from 47 by bridging the tens boundary; +3, +10, +5)





Adjusting

35 – 9 (by subtracting 10 and adding 1)
35 –19 (by subtracting 20 and adding 1)

Using known facts and Place Value

57 – 4
7 – 4 = 3 so 57 – 4 = 53

Estimating

check calculations by subtracting in a different way
solve 16 – 9 by 16 – 10 + 1
check by counting up from 9 to 16

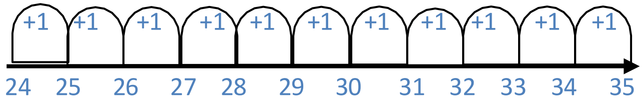
Subtract 1 and 2-digit numbers to 100 (also applicable in year 3)

34 – 3 = 31
Begins without crossing the 10s boundary

Number Line Subtraction

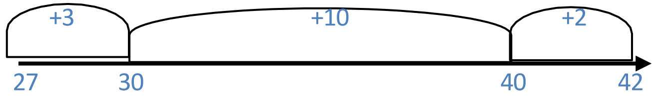
Using complementary addition to subtract by counting on in ones and tens through the use of a number line, then adding up the "jumps".

35 – 24 = 11

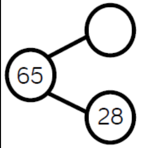


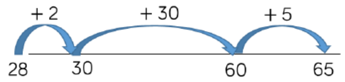
Encourage use of known number bonds to get to multiples of 10, and from there jump in 10s.

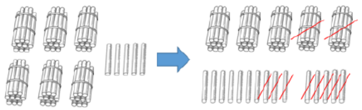
42 - 27

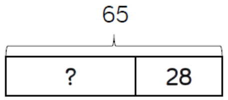


10 + 3 + 2 = 15

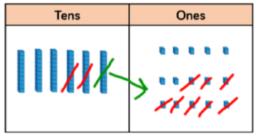




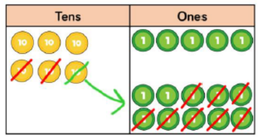




$$\boxed{65 - 28 = 37}$$



$$\begin{array}{r} {}^{5}\cancel{6}{}^{1}5 \\ -\ 28 \\ \hline 37 \\ \hline \end{array}$$



SUBTRACTION: Y3

Understanding the operation and related vocabulary.

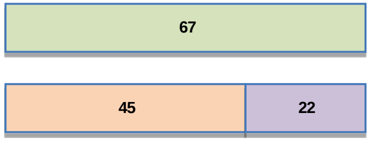
Understanding the operation

Understand that the principles of the commutative and associative laws do not apply to subtraction

Recognise that 41-35 is different from 35-41

Recognise that if calculating 19-6-3 the order matters (we cannot calculate 6-3 first)

Understand the inverse relationship between addition and subtraction



Write the related number sentences

45+22=67 22+45=67 67=45+22 67=22+45

67-22=45 67-45=22 22=67-45 45=67-22

Solve missing number problems e.g.

62- ☐ =19 ☐ =68-54 ☐ -☐ =25

59+34 = 100 - ☐ 45 < ☐ - 6 ☐ - ☐ > 54 + 9

Vocabulary

Hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange

See also Y1 and Y2

Generalisations

Noticing what happens to the digits when you count in tens and hundreds.

Odd – odd = even etc (see Year 2)

Inverses and related facts – develop fluency in finding related addition and subtraction facts.

Mental Calculations

Number facts

Continue to recall and use subtraction facts to 20 fluently, and derive and use related facts beyond 100 using vocabulary related to subtraction

16 subtract 9, 150 minus 70, the difference between 80 and 170, 30 fewer than 110

Know pairs of two-digit numbers with a total of 100 and derive related subtraction facts e.g.

100-79, 100-43, 100-12 ….

Mental methods and jottings

subtract numbers mentally, including:

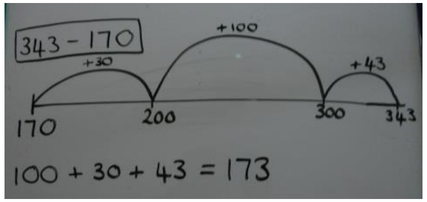
* a three-digit number and ones
* a three-digit number and tens
* a three-digit number and hundreds

Counting up

102 – 97 (by counting up from 97, bridging the hundreds boundary; +3, +2)

With jottings

343 – 170 (by counting up from 170, bridging the hundreds boundary; +30, +100, +43)



Adjusting:

234 – 99 (by subtracting 100 and adding 1)

With Jottings:

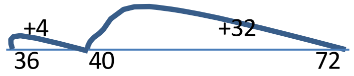
387 – 59 (by subtracting 60 and adding 1)

Written Calculations

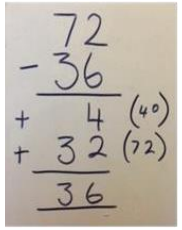
Continue to use number lines to show the difference with 3 digit numbers but begin to prepare for decomposition, using smaller numbers to begin with.

Complementary addition to subtract

72 - 36



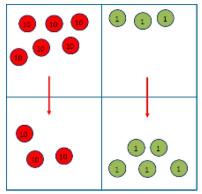
	32
+	4
	36



Expanded decomposition

Introduce expanded column subtraction with no decomposition, modelled with place value counters (Dienes could be used for those who need a less abstract representation) when the child is ready for this method.

98 - 35



	90	8
-	30	5
	60	3

For some children this will lead to exchanging, modelled using place value counters or dienes

Develop the knowledge that the inverse relationship can be used as a checking method.

Key Questions

What do you notice? What patterns can you see?

When comparing two methods alongside each other: What's the same? What's different?

Estimating

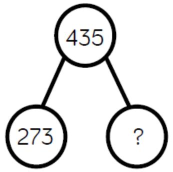
Estimate the answer to a calculation

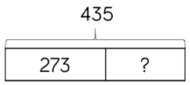
163-48 is approximately 150-50

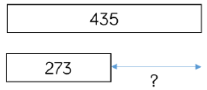
Use inverse operations to check answers

check 102-97=5 with 97+5=102

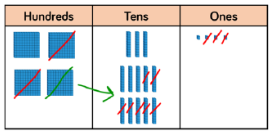
Subtract numbers with up to 3 digits



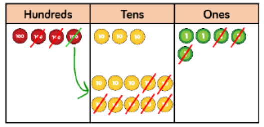




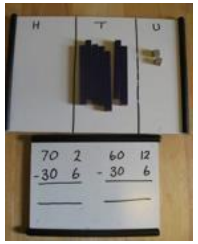
435 − 273 = 262



$$\begin{array}{r} {}^{3}4\,{}^{1}35 \\ -\;273 \\ \hline 262 \end{array}$$



72 – 36



SUBTRACTION: Y4

Understanding the operation and related vocabulary.

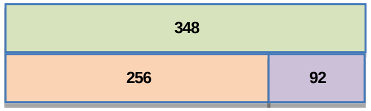
<u>Understanding the operation</u>

Continue to understand that the principles of the commutative and associative laws do not apply to subtraction

recognise that 92-56 is different from 56-92

recognise that if calculating 73-27-8 the order matters (we cannot calculate 27-8 first)

Continue to understand the inverse relationship between addition and subtraction



Write the related number sentences

256+92=348   92+256=348   348=256+92   348=92+256

348-256=92   348-92=256   92= 348-256   256=348-92

Continue to solve missing number problems e.g.

589+318 = 1000 - ☐   450 < ☐ - 60   ☐ - ☐ > 345+199

<u>Vocabulary</u>

add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make..? how much more? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? Equals sign, is the same as.

<u>Generalisations</u>

Investigate when re-ordering works as a strategy for subtraction. eg. 20 – 3 – 10 = 20 – 10 – 3, but 3 – 20 – 10 would give a different answer.

<u>Some Key Questions</u>

What do you notice?

Mental Calculations

<u>Number facts</u>

Continue to use knowledge of subtraction facts and place value to derive related facts using subtraction vocabulary

8000 subtract 3000, 1700 minus 800, the difference between 700 and 1400, 300 fewer than 1200

Know complements to the next multiple of 100 e.g.

367 + ☐ = 400   739 + ☐ = 800

<u>Mental methods and jottings</u>

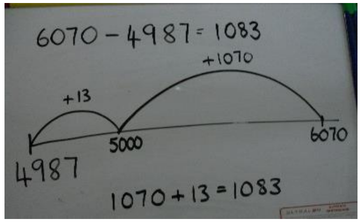
Continue to practise mental methods of subtraction with increasingly large numbers.

<u>Counting Up:</u>

607 – 288 (by counting up from 288, bridging the hundreds boundary; +12, +7)

With jottings:

6070 – 4987 (by counting up from 4987, bridging the thousands boundary; +13, +1070)



<u>Adjusting (with jottings):</u>

1487 – 199 (by subtracting 200 and adding 1)

<u>Estimating</u>

Estimate the answer to a calculation

3062-2581 is approximately 3000-2500

Written Calculations

By Year 4, most children will be confident using expanded decomposition, with the support of practical equipment and models and images and will be able to use this method for HTU – HTU and ThHTU – ThHTU

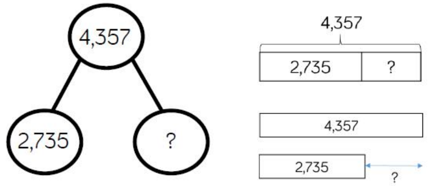
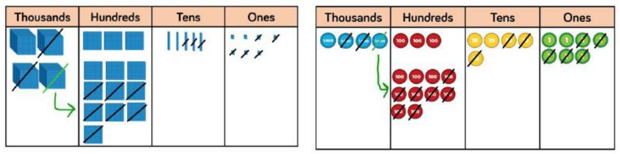
607 – 468 =

<u>Column Subtraction</u>

Subtract numbers with 3 and 4 digit numbers using the formal written method of column subtraction <u>where appropriate</u>

Subtract decimals to 2 decimal places (in the context of money or measures)

607 – 468

What’s the same? What’s different? Can you convince me? How do you know?	Use inverse operations to check answers check 564 – 150= 414 with 414+150=564 **<u>Subtract numbers with up to 4 digits</u>**  3 1 4357 – 2735 1622 4,357 – 2,735 = 1,622  	9 5 6 10 17 - 4 6 8 1 3 9 Use complementary addition to help subtraction if children need additional support with 4 digit numbers. (see Y3 for an example)

SUBTRACTION: Y5

Understanding the operation and related vocabulary.

Understanding the operation

Continue to solve missing number problems

6.5-☐=2.3 ☐=3-0.8 ☐-☐=1.2

5.4+2.7 = 10.3 - ☐ 5.2 < ☐ - 0.9 ☐ - ☐ > 7.2-1.9

Begin to use brackets

(10-3) x 6 = ☐ 10 - (0.5 x 7) = ☐

Vocabulary

tens of thousands boundary,

Also see previous years

Generalisation

Sometimes, always or never true? The difference between a number and its reverse will be a multiple of 9. What do you notice about the differences between consecutive square numbers?

Investigate a – b = (a-1) – (b-1) represented visually.

Key Questions

What do you notice?

What's the same? What's different?

Can you convince me?

How do you know?

Mental Calculations

Number facts

Continue to use knowledge of subtraction facts and place value to derive related facts with numbers to one decimal place (using subtraction vocabulary)

1.2 subtract 0.7, 1.8 minus 0.9, the difference between 2 and 1.3, 0.3 fewer than 1.7

Know complements to 1

0.78 + ☐ = 1 0.52 + ☐ = 1

Recall pairs of three-digit numbers with a total of 1000 and derive related subtraction facts

1000-453, 1000-239, 1000-712 …

Mental methods and jottings

Subtract numbers mentally with increasingly large numbers

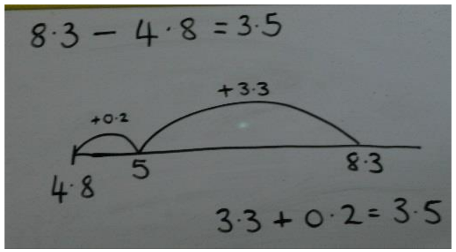
Subtract tenths, and one-digit whole numbers and tenths

Counting up:

7.2 – 6.8 (by counting up from 6.8 by bridging the units boundary; +0.2, +0.2)

With jottings:

8.3 – 4.8 (by counting up from 4.8 by bridging the units boundary; +0.2, +3.3)



Adjusting (with jottings):

8.3 – 1.9 (by subtracting 2 and adding 0.1)

Written Calculations

Subtract whole numbers with up to 5 digits using formal written methods

Subtract decimals, including a mix of whole numbers and decimals and decimals with different numbers of decimal places up to 2 decimal places.

Expanded subtraction

5642 – 2861

4000	1500		
~~5000~~	~~600~~	140	2
- 2000	800	60	1
2000	700	80	1

= 2781

Decomposition:

Some children may still need the support of practical apparatus and models and images, though most children will no longer need to use the expanded method and should be confident using the compact method, having an understanding of the value of each digit. They should be confident using this method for decimals too.

25 034 – 7185 36.25 – 14.6

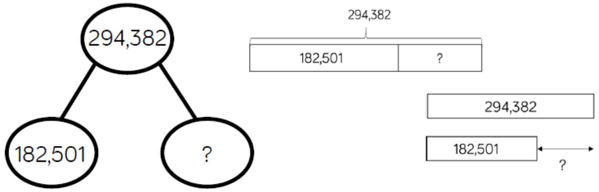
(See year 4 for an example)

Estimating

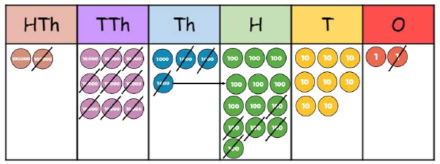
Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy

25 034 – 7185 is approximately 25 000 – 7000

Subtract numbers with more than 4 digits (also applicable in year 6)

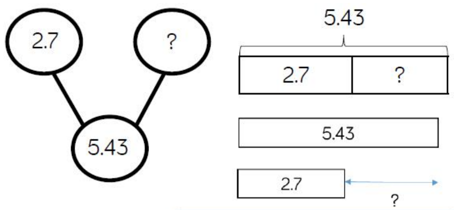


294,382 − 182,501 = 111,881



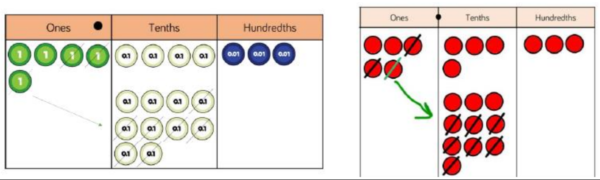
	2	9	³~~4~~	¹3	8	2
−	1	8	2	5	0	1
	1	1	1	8	8	1

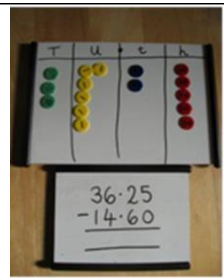
Subtract with up to 3 decimal places

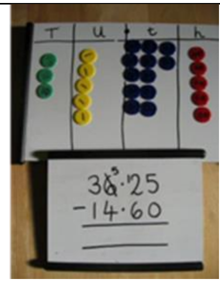


$$\begin{array}{r} {}^{4}\cancel{5}.{}^{1}43 \\ -\ 2.7 \\ \hline 2.73 \end{array}$$

5.43 − 2.7 = 2.73







Use complementary addition to help subtraction if children need additional support with 5 digit numbers. (see Y3 for an example)

SUBTRACTION: Y6

Understanding the operation and related vocabulary.

Understanding the operation
Use knowledge of the order of operations
Understand that when there are no brackets in an expression, do multiplication or division before addition or subtraction
Understand that if the operations are at the same level of priority, work out the example from left to right

Continue to solve missing number problems
0.63-☐=0.32   ☐=0.5-0.33   ☐-☐=0.11
0..89 – 0.4 = 1.3 - ☐   0.75 < ☐ - 0.06   ☐ - ☐ > 0.82 – 0.09

Explore the order of operations using brackets
compare 14 – (3 + 5) with (14 – 3) + 5

Vocabulary
See previous years

Generalisations
Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as PEMDAS, or could be encouraged to design their own ways of remembering.
Sometimes, always or never true? Subtracting numbers makes them smaller.

Some Key Questions
What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?

Mental Calculations

Number facts
Continue to use knowledge of subtraction facts and place value to derive related facts with numbers to two decimal places (using subtraction vocabulary)
3.09 subtract 0.04, 0.16 minus 0.08, the difference between 0.2 and 0.12, 0.06 fewer than 0-19

Know complements to the next whole number
4.83 + ☐ = 5   7.125 + ☐ = 8

Mental methods and jottings
Perform mental calculations, including with mixed operations, large numbers and decimals

Calculate intervals across zero e.g.
the drop in temperature from +5 to -3

Counting on using number lines (with jottings):
6.14 – 5.76 (by counting up from 5.76 by bridging the units boundary; +0.24, +0.14)

Adjusting (with jottings):
7.65 – 0.99 (by subtracting 1 and adding 0.01)

Estimating:
Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.
60 .31 – 17.884 is approximately 60-18

Written Calculations

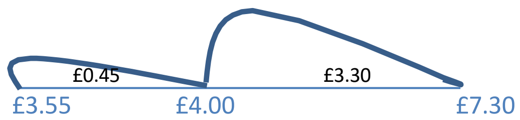
DECOMPOSITION:

By this stage, children should be confident using the compact method, for 6 digit numbers and decimals up to 3 places.

500 203 – 34 456   60.31 – 17.884

Use complementary addition to help subtraction if children need additional support with larger numbers and problems involving money.

£7.30 - £3.55



£3.30 + £0.45 = £3.75

PROGRESSION MAP
Multiplication

This must be viewed alongside the division map so that connections can be made.

YR	Y1	Y2	Y3	Y4	Y5	Y6
Understanding the operation and related vocabulary						
	Begin to understand multiplication by using concrete objects, pictorial representations and arrays to solve problems; make connections between the different representations	Understand multiplication • repeated addition • describing an array • scaling (to compare 2 items) e.g. twice as high • correspondence problems – one to many	Understand multiplication as • repeated addition • describing an array • scaling • correspondence problems – one to many and many-to-many	Continue to understand multiplication as • repeated addition • describing an array • scaling • correspondence problems – one to many and many-to-many	Understand • scaling by simple fractions • simple rates	Continue to understand • scaling by fractions • rate
		show that multiplication of two numbers can be done in any order	understand commutativity and associativity	understand the distributive law continue to understand commutativity and associativity	continue to understand the distributive, commutative and associative laws	use their knowledge of the order of operations
	.	recognise the inverse relationship between multiplication and division	understand the inverse relationship between multiplication and division	continue to understand the inverse relationship between multiplication and division		
record using marks that they can interpret and explain	use pictorial representations	write mathematical statements using the multiplication (×), and equals (=) signs	solve missing numbers problems involving multiplication	continue to solve missing number problems	continue to solve missing number problems begin to use brackets	continue to solve missing number problems explore the order of operations using brackets
begin to use the vocabulary involved in multiplying *double, pattern*	begin to use the vocabulary involved in multiplying *array, row, column, groups of, lots of,*	understand and use the vocabulary involved in multiplying *multiple, multiply, table, times, once, twice, three, ten .... times as big, repeated addition*	understand, read and spell vocabulary related to multiplication correctly *product*	understand, read and spell vocabulary related to multiplication correctly *factor*	read, spell and pronounce mathematical vocabulary related to multiplication correctly *square, cube, prime numbers, prime factors, composite numbers, common factor*	read, spell and pronounce mathematical vocabulary related to multiplication correctly *common multiple*
Recalling number facts						

begin to count in twos and tens	count in multiples of twos, fives and tens	count in steps of 2, 3, and 5 from 0	count from 0 in multiples of 4, 8, 50 and 100	count in multiples of 6, 7, 9, 25 and 1000	use knowledge of counting in multiples to count in decimal steps (one decimal place)	use knowledge of counting in multiples to count in decimal steps (two decimal places)
know doubles of all numbers to 5	know doubles of all numbers to 10	recall doubles of all numbers to 15 and doubles of multiples of 5 to 50	recall doubles of all numbers to 20, doubles of multiples of 5 to 100 and doubles of multiples of 100 to 500	derive doubles of multiples of 50 to 1000 and multiples of 1000	derive doubles of decimals (to one decimal place) using knowledge of place value	derive doubles of decimals (to two decimal places) using knowledge of place value
	begin to recognise odd and even numbers	recall and use multiplication facts for the 2, 5 and 10 multiplication tables recognise odd and even numbers	recall and use multiplication facts for the 3, 4 and 8 multiplication tables and begin to use knowledge of place value to derive related facts	recall multiplication facts for multiplication tables up to 12 × 12, and use place value to derive related facts	continue to recall multiplication facts for multiplication tables up to 12 × 12 fluently, and derive and use related facts	continue to recall multiplication facts for multiplication tables up to 12 × 12 fluently, and derive and use related facts
				recognise and use factor pairs	identify multiples and factors, and common factors of two numbers. establish whether a number up to 100 is prime and recall primes up to 19; find prime factors recognise and use square and cube numbers	identify common factors, common multiples and prime numbers continue to use square and cube numbers
Mental methods and mental methods with jottings						
count a set of objects by grouping in 2s solve simple problems involving doubling and equal groups	count a set of objects by grouping in 2s, 5s or 10s solve problems involving doubling and equal groups	calculate mathematical statements for multiplication within the multiplication tables through mental addition and number line jottings	calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers use doubling to connect 2, 4 and 8 multiplication tables	multiply mentally using place value, known and derived facts, including: multiplying by 0 and 1; multiplying together three numbers	multiply numbers mentally drawing upon known facts use factors to construct equivalence statements begin to multiply tenths, and one-digit whole numbers and tenths by one-digit whole numbers	perform mental calculations, including with mixed operations, large numbers and decimals
Formal written layout						
			begin to use formal written methods for two-digit numbers multiplied by one-digit numbers (for known multiplication facts)	multiply two-digit and three-digit numbers by a one-digit number using formal written layout	multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers	multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication

					multiply numbers with up to one decimal places by a one-digit whole number	multiply numbers with up to two decimal places by one-digit and two-digit whole numbers
Estimating and checking						
		begin to use equivalent calculations to check answers	estimate the answer to a calculation use inverse operations to check answers use equivalent calculations to check answers	estimate the answer to a calculation use inverse operations to check answers use equivalent calculations to check answers	use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy continue to use appropriate strategies to check answers	use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy. continue to use appropriate strategies to check answers

MULTIPLICATION: Y1

Understanding the operation and related vocabulary

Understanding the operation

Begin to understand multiplication by using concrete objects, pictorial representations and arrays to solve problems; make connections between the different representations.
begin to use the vocabulary involved in multiplying

Vocabulary

ones, groups, lots of, doubling
repeated addition array, row, column,
groups of, lots of, times, columns, rows
longer, bigger, higher etc
times as (big, long, wide …etc)

Generalisations

Understand 6 counters can be arranged as 3+3 or 2+2+2

Understand that when counting in twos, the numbers are always even.

Some Key Questions

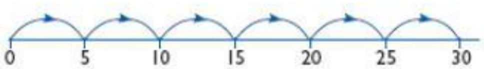
Why is an even number an even number?
What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?

Mental Calculations

Number facts

Count in multiples of twos, fives and tens
0 2 4 6 8 10…

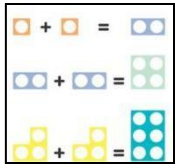
How many legs have 5 teddies got altogether?



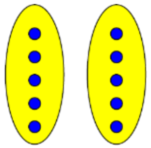
How much money have I got in my purse? 5,10,15,20

How many 10ps do I need to buy a chocolate bar for 30p?

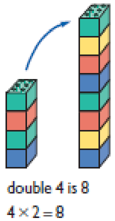
Know doubles of all numbers to 10



Double 3 is ☐ 8+8=☐
Double 5 is ☐ 6+6=☐

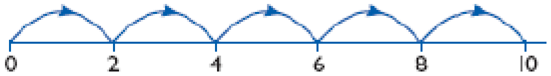


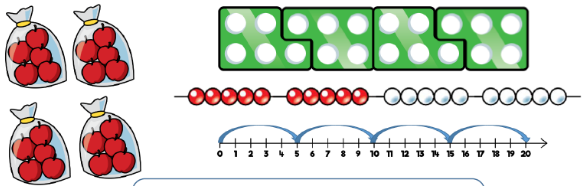
2 groups of 5
How many altogether?
5+5
Double 5



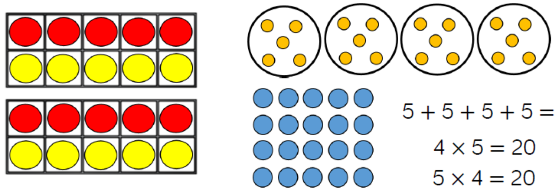
Written Calculations

No formal written layout.
Children will be recording their mathematics using pictorial representations, arrays, number lines and mathematical statements.





One bag holds 5 apples.
How many apples do 4 bags hold?



$5 + 5 + 5 + 5 = 20$
$4 \times 5 = 20$
$5 \times 4 = 20$

Also applicable in year 2

Begin to recognise odd and even numbers

Use cubes to make 9 and recognise it is odd (as the cubes cannot be paired)

Sort Numicon into odd and eve numbers

What happens if we out two odd numbers together?

Mental Methods and jottings

Counting

Count a set of objects by grouping in 2s, 5s or 10s

Count these marbles (2 at a time)

Solve problems involving doubling and equal groups

I need 5 eggs to bake a cake. How many eggs will I need to bake 2 cakes?

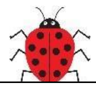
Counting on

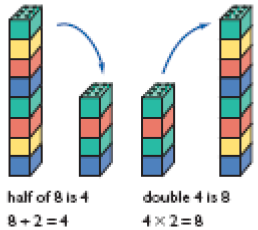
There are 3 pots. Each pot has 2 seeds in. How many seeds are planted?

(by counting on in twos using objects or pictures to keep track)

Doubling and halving

A ladybird has 6 spots on each wing. How many spots are there altogether?



	(by recognising 6+6=12) 	

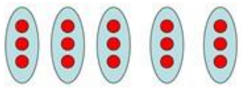
MULTIPLICATION: Y2

Understanding the operation and related vocabulary

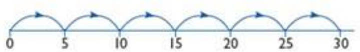
Understanding the operation

Understand multiplication as

- repeated addition
- describing an array
- scaling (to compare 2 items) e.g. twice as long
- correspondence problems – one to many



5 groups of 3
3+3+3+3+3=15



5+5+5+5+5+5=30
5x6=30
5 multiplied by 6
6 groups of 5

3 groups of 10 pencils
10+10+10=30
10x3=30

Show that multiplication of two numbers can be done in any order

recognise that 5 x 3 is equal to 3 x 5

Mental Calculations

Number facts

Count in steps of 2, 3, 5 and 10 from 0

0 3 6 9 12 15 18 30
50 45 40 35 30 0

Recall doubles of all numbers to 15 and doubles of multiples of 5 to 50

Double 13 is □   11+11=□
Double 25 is □   45+45=□

Recall and use multiplication facts for the 2, 5 and 10 multiplication tables

3 groups of 10   multiply 7 by 2   5 multiplied by 4

Recognise odd and even numbers

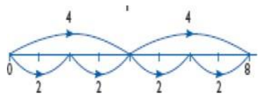
Explain why 27 is an odd number

Mental Methods and Jottings

Written Calculations

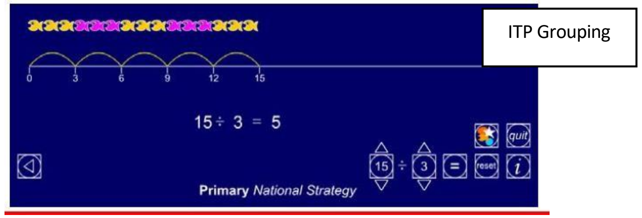
No formal written layout.
Children will be recording their mathematics using pictorial representations, arrays, number lines and mathematical statements.

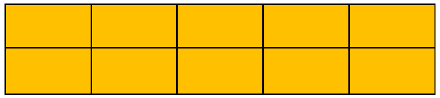
5 groups of 3=15
3 groups of 5=15
3+3+3+3+3=15
5+5+5=15
3x5=15
5x3=15



2 groups of 4=8
4 groups of 2=8
4x2=8
2x4=8

Recognise the inverse relationship between multiplication and division





Write the related number sentences
5x2=10 2x5=10 10=5x2 10=2x5
10÷2=5 10÷5=2 2=10÷5 5=10÷2

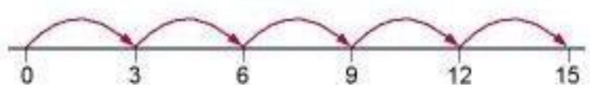
Write mathematical statements using the multiplication (×), and equals (=) signs
5x4=20 16=8x2
3x☐=15 ☐=2x7 20=☐x☐

Vocabulary
multiple, multiply, multiplication array, multiplication tables / facts, groups of, lots of, times, columns, rows, once, twice, three, ten times as big, repeated addition,

Generalisation

Counting on/repeated addition
7x5 (by counting on in fives using fingers to keep track)

With jottings 3x5 (by counting on in threes using a number line to keep track)



Doubling and halving
7x2 (by recalling the doubles fact)

With partitioning
12x2 (by doubling 10, doubling 2 and recombining)

12
10 2
double
20 4
24

Estimating and Checking
Begin to use equivalent calculations to check answers

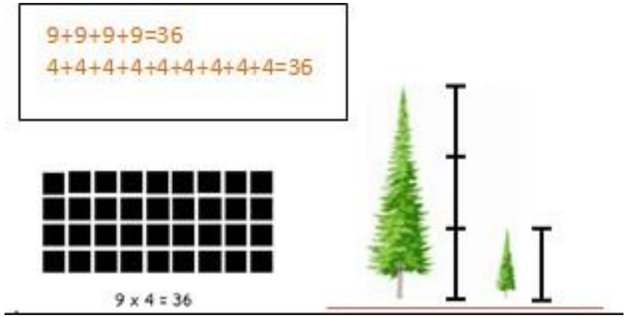
Repeated addition can be shown mentally on a number line Inverse relationship between multiplication and division. Use an array to explore how numbers can be organised into groups. **<u>Some Key Questions</u>** What do you notice? What's the same? What's different? Can you convince me? How do you know?		

MULTIPLICATION: Y3

Understanding the operation and related vocabulary

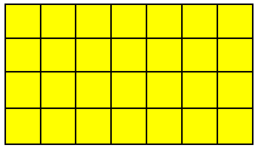
Understanding the operation as

- repeated addition
- an array
- scaling – comparison and enlargement
- correspondence problems – one to many and many-to-many



Understand commutativity and associativity

recognise that 7x4 is equal to 4x7



recognise that if calculating 2x3x10 the numbers can be combined in any order

Understand the inverse relationship between multiplication and division

6x3=18 3x6=18 18=6x3 18=3x6

18÷3=6 18÷6=3 3=18÷6 6=18÷3

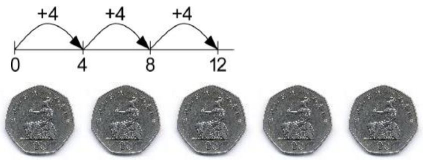
Mental Calculations

Number facts

Count from 0 in multiples of 4, 8, 50 and 100

0 8 16 24 32 ....

500 450 400 350 ....



50,100,150,200, 250

Recall doubles of all numbers to 20, doubles of multiples of 5 to 100 and doubles of multiples of 100 to 500

Double 17 is ☐ 19x2=☐

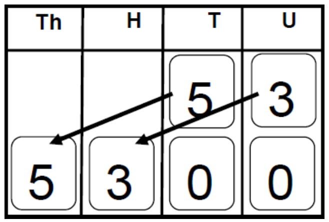
Double 65 is ☐ 85x2=☐

Double 300 is ☐ 400+400=☐

Recall and use multiplication facts for the 2, 3, 4, 5, 8 and 10x multiplication tables and begin to use knowledge of place value to derive related facts using correct vocabulary

3 groups of 8 multiply 9 by 4

the product of 8 and 4 50 multiplied by 4



Place value cards

Use digit cards to make numbers in the grid. Show how each digit in a number moves one column to the left when a number is multiples by 10 and two columns to the left when a number is multiplied by 100.

Mental Methods and Jottings

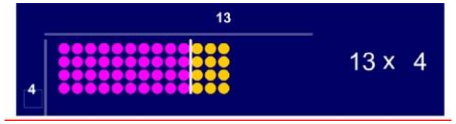
Counting on

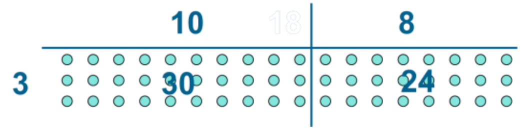
5x14 (by counting on in fives from 50)

Written Calculations

Begin to use formal written methods for two-digit numbers multiplied by one-digit numbers (for known multiplication facts)

Use models and images to demonstrate grid method





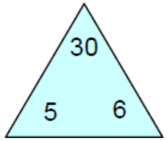
Use manipulatives such as Dienes apparatus and place value counters

38x5

x	30	8
5	150	40

190

***Children to use grid method <u>by the end</u> of year 3**



5 × 6 = 30
6 × 5 = 30
30 ÷ 6 = 5
30 ÷ 5 = 6

Solve missing numbers problems involving multiplication

3x☐=15   ☐=2x7   20=☐x☐

25 + 10 = 5 x☐   15<☐ x 2☐x☐> 20

Vocabulary

partition, grid method, inverse, product

Generalisations

Connecting x2, x4 and x8 through multiplication facts

Comparing times tables with the same times tables which is ten times bigger. If 4 x 3 = 12, then we know 4 x 30 = 120. Use place value counters to demonstrate this.

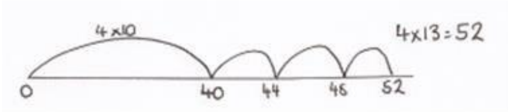
When they know multiplication facts up to x12, do they know what x13 is? (i.e. can they use 4x12 to work out 4x13 and 4x14 and beyond?)

Key Questions

What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?

with jottings

4x13 (by counting on in fours from 4x10 using a number line to keep track)



Partioning (with distributive law)

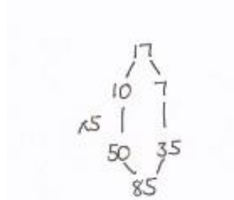
Without crossing the tens boundary

32x3= (30x3=90, 2x3=6, 90+6=96)

with jottings

Crossing the tens boundary

17x5= (10x5=50, 7x5=35, 50+35=85)



Doubling and halving

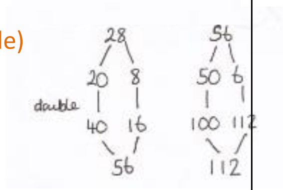
9x20 (multiply by 10 and then double)

9x10=90   Double 90 is 180

with jottings

28x4 (double and double again)

Double 28 is 56, double 56 is 112



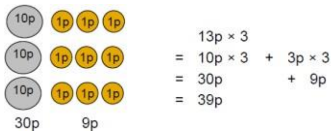
Using known facts and place value

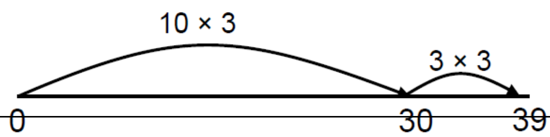
Use manipulatives to demonstrate this.

4x11

4x10=40 so 4x11=44

13x3

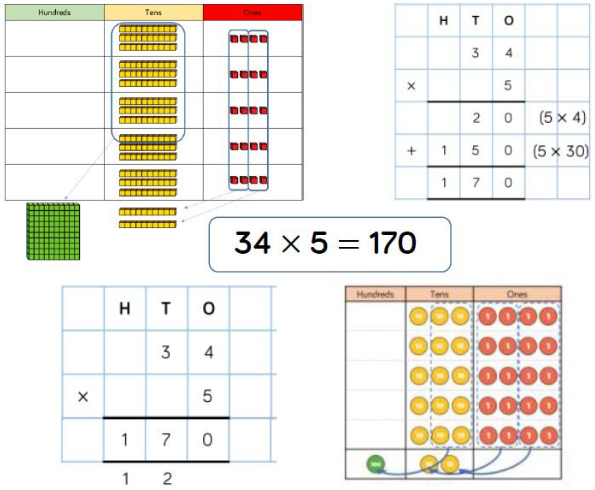




30x5
3x5=15 so 30x5=150

<u>Estimating and checking</u>
Estimate the answer to a calculation
38x5 is approximately 40x4
Use inverse operations and equivalent calculations to check answers.
28x4 by doubling (28x2x2) or using partitioning (20x4 and 8x4)

<u>Multiplying 2-digit numbers by 1-digit numbers</u> <u>(also applicable in year 4) – pictorial leading to written</u>



MULTIPLICATION: Y4

Understanding the operation and related vocabulary

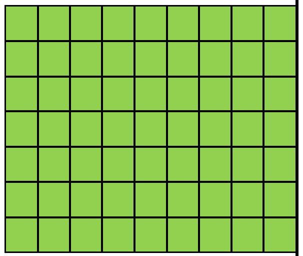
Understanding the operation of multiplication as:

- repeated addition
- describing an array
- scaling – comparison and enlargement
- correspondence problems – one to many and many-to-many

11+11+11+11=44
7+7+7+7+7+7+7=49

Understand the distributive law
recognise that 14x5 is the same as 10x5 added to 4x5
36 x 9 = (30 x 9) + (6 x 9) = 270 + 54 = 324

continue to understand commutativity and associativity
7x9 is equal to 9x7
4x8x10 can be combined in any order:
4 x 8 x 10 = 320
8 x 10 x 4 = 320
10 x 4 x 8 = 320



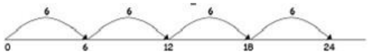
Continue to understand the inverse relationship between multiplication and division
6x7=42 7x6=42 42=6x7 42=7x6
42÷7=6 42÷6=7 7=42÷6 6=42÷7

Solve missing numbers problems involving multiplication
3x☐=15 ☐=2x7 20=☐x☐
25 + 10 = 5 x☐ 15<☐ x 2☐x☐> 20

Vocabulary
Factor

Mental Calculations

Number facts
Count in multiples of 6, 7, 9, 25 and 1000
0 7 14 21 28 …
300 275 250 225 200 …



25	50			
100				
	325		375	

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

Derive doubles of multiples of 50 to 1000 and multiples of 1000
Double 950 is ☐ 750x2=☐
Double 8000 is ☐ 6000+6000=☐

Recall multiplication facts for multiplication tables up to 12 × 12, and use place value to derive related facts (using subtraction vocabulary)
7 groups of 8 multiply 9 by 6
the product of 8 and 11 60 multiplied by 4

Recognise factor pairs

Mental Methods and Jottings
Multiply mentally using place value, known and derived facts, including: multiplying by 0 and 1; multiplying together three numbers and multiplying decimals by 10 and 100.

Counting on
3x42 (by counting on in threes from 120)

Written Calculations

Multiply two-digit and three-digit numbers by a two and three digit number using formal written layout

Children to embed and deepen their understanding of the grid method to multiply. Ensure this is still linked back to their understanding of arrays and place value counters.

36 x 4 = 144

X	30	6
4	120	24

120 + 24 = 144 (add the partial products)

127 x 6 = 762

x	100	20	7
6	600	120	42

600 + 120 + 42 = 762 (add the partial products)

Generalisations

When they know multiplication facts up to x12, do they know what x13 is? (i.e. can they use 4x12 to work out 4x13 and 4x14 and beyond?)

Key Questions

What do you notice?

What's the same? What's different?

Can you convince me?

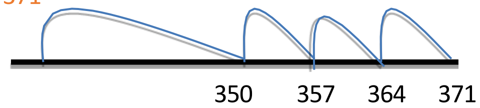
How do you know?

With jottings

7x53 (by counting on in sevens from 7x50 using a number line to keep track)

3 X 50

3 X 53 = 371



Partitioning (using the distributive law)

53x6 (50x6=300 3x6=18 300+18=318)

Using doubling and halving

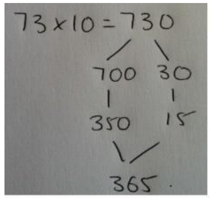
35x8 (double, double and double again)

Double 35 is 70, double 70 is 140, double 140 is 280

With jottings

73x5 (multiply by 10 and then halve)

73x10=730 Half of 730 is 365 (Some children may need to partition 730 in a different way)



Using factors

15x6 = 15x3x2

15x3=45 45x2=90

Using known facts and place value

24x10=240 so 24x9=216 (by subtracting 24 from 240)

800x6

8x6=48 so 800x6=480

36 x 42

x	30	6
40	1200	240
2	60	12

1200 + 240 + 60 + 12 = 1512

127 x 23

X	100	20	7
20	2000	400	140
3	300	60	21

2000 + 400 + 300 + 140 + 60 + 21 = 2921

Higher attainers move onto expanded menthod for multiplication.

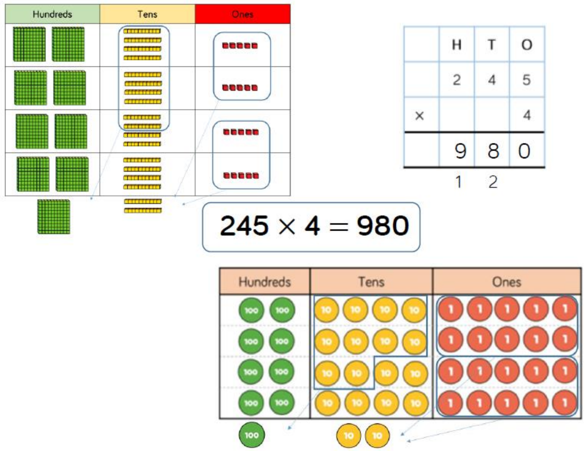
36 x 4

```
   36
X   4
   24   (4 x 6)
  120   (4 x 30)
  144
```

Linked to money

£3.36 x 2

```
£   3.36
X      2
     .12  (2 x .06)
     .60  (2 x .30)
    6.00  (2 x 3.00)
£   6.72
```

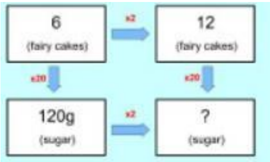
	<u>Multiplying 3-digit numbers by 1-digit numbers</u><u>– pictorial leading to written</u>  	

MULTIPLICATION: Y5

Understanding the operation and related vocabulary

<u>Understanding the operation of multiplication as:</u>

- scaling by simple fractions
- simple rates

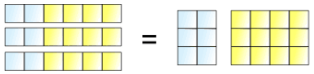


Continue to understand the distributive, commutative and associative laws

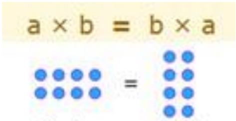
recognise that 37x6 is the same as 30x6 added to 7x6 (distributive)

recognise that 25x7 is equal to 7x25 (commutative)

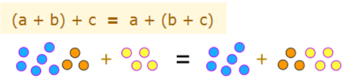
recognise that if calculating 18x4x10 the numbers can be combined in any order (associative)



3 x (2+4)   3x2 + 3x4



2 × 4   4 × 2



(6 + 3) + 4   6 + (3 + 4)

Continue to understand the inverse relationship between multiplication and division

write the related number sentences

6x0.7=4.2 0.7x6=4.2 4.2=6x0.7 4.2=0.7x6

4.2÷0.7=6 4.2÷6=0.7 0.7=4.2÷6 6=4.2÷0.7

Mental Calculations

<u>Number facts</u>

Use knowledge of counting in multiples to count in decimal steps (one decimal place)

0.6 1.2 1.8 2.4 ....

8.4 7.7 7.0 6.3 ...

Derive doubles of decimals (to one decimal place) using knowledge of place value

Double 0.4 is ☐   0.7x2=☐

Double 3.8 is ☐   5.6+5.6=☐

Continue to recall multiplication facts for multiplication tables up to 12 × 12 fluently, and derive and use related facts

7 groups of 8   multiply 12 by 9

the product of 80 and 40   0.6 multiplied by 4

x	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Identify multiples and factors, and common factors of two numbers.

list the factors of 96

identify the common factors of 30 and 36 by listing factor pairs

give a number that is a multiple of 3 and a multiple of 2 (and recognise these are multiples of 6)

list the multiples of 9 between 150 and 180 (using tests of divisibility)

Written Calculations

Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.

Multiply numbers with up to one decimal place by one-digit whole number.

Use grid method, progressing to short and long multiplication for numbers with more digits when understanding in secure.

<u>Grid method</u>

46 x 82   43.2 x 7

×	30	5
20	600	100
6	180	30

600 + 100 = 700
180 + 30 = 210
700 + 210 = 910

x	6
2.0	12.0
0.3	1.8

13.8

Short multiplication

36 x 4 = 144

```
  30 + 6
 X     4
     2 4    (4 x 6 = 24)
 + 1 2 0    (4 x 30 = 120)
   1 4 4
```

Include an addition symbol when adding partial products.

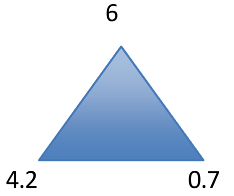
```
36x4=   36
       X 4
     1 4 4
       2
```

Short multiplication for multiplying by a single digit

x	300	20	7
4	1200	80	28

```
    3 2 7
 x      4
  1 3 0 8
    1 2
```

Pupils could be asked to work out a given calculation using the grid, and then compare it to 'your' column method. What are the similarities and differences? Unpick the steps and show how it reduces the steps.



Continue to solve missing number problems

6x☐=540 ☐=0.4x8 480=☐x☐

90x40 = 6x☐ 2.5<☐ x5 ☐x☐> 700x8

begin to use brackets

(10+3) x 7 = ☐ ☐=10 + (0.4 x 8)

Vocabulary

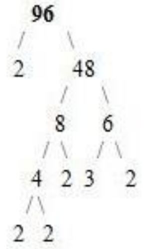
cube numbers
prime numbers
square numbers
common factors
prime number, prime factors
composite numbers

Generalisation

Relating arrays to an understanding of square numbers and making cubes to show cube numbers.
Understanding that the use of scaling by multiples of 10 can be used to convert between units of measure (e.g. metres to kilometres means to times by 1000)

Key Questions

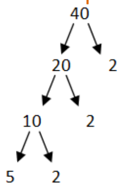
What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?
How do you know this is a prime number?



Establish whether a number up to 100 is prime and recall primes up to 19; find prime factors

explain why 23 is a prime number

list the prime factors of 20 (20 = 2x10 = 2x2x5)



5 x 2 x 2 x 2

Recognise and use square and cube numbers

What is... 8^2? 3^3?

Mental Methods and Jottings

Multiply numbers mentally drawing upon known facts

Use factors to construct equivalence statements

4 x 35 = 2 x 2 x 35

3 x 270 = 3 x 3 x 9 x 10 = 9^2 x 10

begin to multiply tenths, and one-digit whole numbers and tenths by one-digit whole numbers

0.2 x 3 = 0.6

Partitioning (using the distributive law)

1.2x7 (1x7=7 0.2x7=1.4 7+1.4=8.4)

Doubling and halving

3.7x4 (Double and double again)

Double 3.7 is 7.4, double 7.4 is 14.8

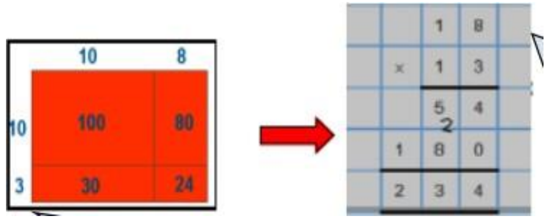
Long multiplication – expanded method

```
   31
X 25
    5 (5x1)
 150(5x30)
   20(20x1)
 600(20x30)
 775
```

Long multiplication

Initially, some children may need to break this down further

```
  3 1          31          31      155
 x2 5         x 5        x 20    + 620
 1 5 5        15 5        620      775
 6 2 0
 7 7 5
```



Using factors

25x12=25x2x6

25x2=50 50x6=300

Using Known facts and place value

13x19

13x20=260 so 13x19=247 (subtract 26 from 260)

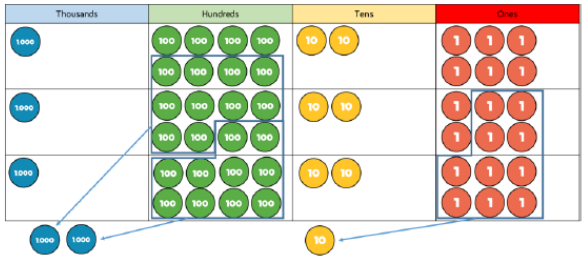
Estimating and Checking

Check 86x9 by using an equivalent calculation

Multiply by 10 and adjust (860-86) or partition (80x9 added to 6x9)

Pictorial examples leading to written method

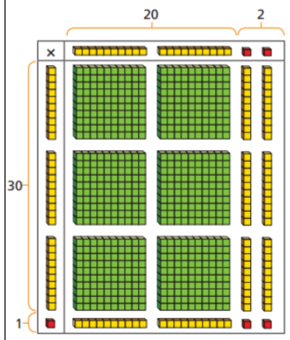
Multiply 4-digit numbers by 1-digit numbers

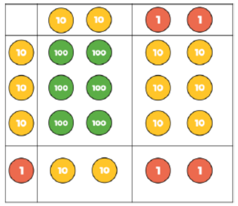


$1{,}826 \times 3 = 5{,}478$

	Th	H	T	O
	1	8	2	6
×				3
	5	4	7	8
	2		1	

Multiply 2-digit numbers by 2-digit numbers



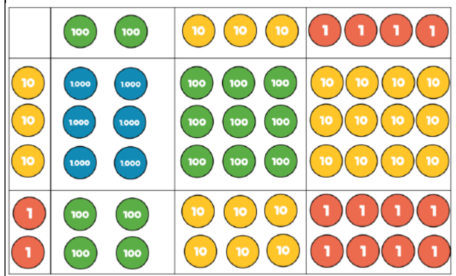


×	20	2
30	600	60
1	20	2

	H	T	O
		2	2
×		3	1
		2	2
	6	6	0
	6	8	2

$22 \times 31 = 682$

Multiply 3-digit numbers by 2-digit numbers



Th	H	T	O
	2	3	4
×		3	2
	4	6	8
$_1$7	$_1$0	2	0
7	4	8	8

×	200	30	4
30	6,000	900	120
2	400	60	8

$234 \times 32 = 7{,}488$

Multiply 4-digit numbers by 2-digit numbers (also applicable in year 6)

TTh	Th	H	T	O
	2	7	3	9
$\times$			2	8
2 $_{2}$	1 $_{5}$	9 $_{3}$	1 $_{7}$	2
5 $_{1}$	4	7 $_{1}$	8	0
7	6	6	9	2
		1		

$2{,}739 \times 28 = 76{,}692$

MULTIPLICATION: Y6

Understanding the operation and related vocabulary

Understanding the operation of multiplication as:

Scaling by fractions

Of the 90 students on a field trip to the zoo, two ninths want to go to see the bears. How many students want to see the bears?

90÷ 2/9 90 ÷9 = 10 10 X 2 = 20

Rate

A car travels 60 miles per hour. How far will it travel in 2 and a quarter hours?

Use their knowledge of the order of operations - BODMAS

Understand that when there are no brackets in an expression, do multiplication or division before addition or subtraction

Understand that if the operations are at the same level of priority, work out the example from left to right

Continue to solve missing number problems

6x☐=0.54 ☐=0.06x8 4.8=☐x☐

0.9x4 = 6x☐ 0.63<☐ x0.09 ☐x☐>0.07x8

Explore the order of operations using brackets

compare 14 ÷ (2 x 5) with (14 ÷ 2) x 5

Vocabulary

common factor/multiple

Generalisations

Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic to remember this or could be encouraged to design their own ways of remembering.

Understanding the use of multiplication to support conversions between units of measurement.

Mental Calculations

Number facts

Use knowledge of counting in multiples to count in decimal steps (two decimal places)

0.09 0.18 0.27 0.36 ....

0.48 0.44 0.4 0.36 ...

Derive doubles of decimals (to two decimal places) using knowledge of place value

Double 0.47 is ☐ 0.73x2=☐

Double 3.08 is ☐ 2.59+2.59=☐

Continue to recall multiplication facts for multiplication tables up to 12 × 12 fluently, and derive and use related facts

30 multiplied by 800 multiply 0.12 by 6

the product of 0.08 and 4 0.4 multiplied by 0.5 identify

common factors, common multiples and pri + r numbers

find the highest common factor of 18 and 24

find the lowest common multiple of 6 and 15

identify whether 87 is a prime number

list the prime factors of 84 (84 = 2x42 = 2x2x21 = 2x2x3x7)

use the tests of divisibility to identify factors and multiples

continue to use square and cube numbers

What is…12^2? 6^3?

Mental Methods and Jottings

Perform mental calculations, including with mixed operations, large numbers and decimals

Partitioning (using distributive law)

6.04x3 (6x3=18 0.04x3=0.12 18+0.12=18.12)

Doubling and halving

Written Calculations

Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication

Multiply numbers with up to two decimal places by one-digit and two-digit whole numbers

GRID METHOD

15.76 x 3

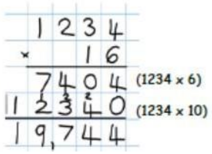
E.g. 15.76 x 3 =

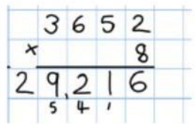
X	10.00	5.00	0.70	0.06
3	30.00	15.00	2.10	0.18

30.00
15.00
2.10
0.18

47.28

Children should not be taught the following method shown below until they are thoroughly secure with mental calculation strategies, recall of multiplication tables and the application of Place Value (see year 5) Develop year 5 methods with more complex calculations such as





<u>Key Questions</u> What do you notice? What's the same? What's different? Can you convince me? How do you know?	0.24x40 (double and double again, then multiply by 10) Double 0.24 is 0.48, double 0.48 is 0.96, 0.96x10=9.6 68x25 (multiply by 100, then halve and halve again) 68x100=6800 Half of 6800 is 3400 Half of 3400 is 1700 <u>Using factors</u> 1.5x16=1.5x2x8 1.5x2=3 3x8=24 32x24 = 32x3x8 32x3=96 96x8=800-(4x8)=768 <u>Using known facts and place value</u> 17x98 17x100=1700 so 17x98 is 1666 (subtract 17x2 from 1700) **<u>Estimating and checking</u>** Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy. 5872x54 is approximately 6000x50 Continue to use appropriate strategies to check answers Check 496x5 by using an equivalent calculation Multiply by 10 and halve or use a known fact and adjust (500x5) –(4x5)	

PROGRESSION MAP
Division

This must be viewed alongside the division map so that connections can be made.

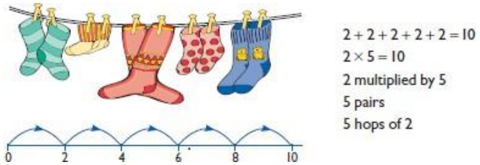
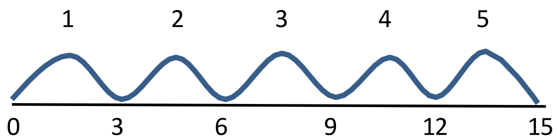
YR	Y1	Y2	Y3	Y4	Y5	Y6
Understanding the operation and related vocabulary						
	Begin to understand division as grouping and sharing by using concrete objects, pictorial representations and arrays to solve problems; make connections between the different representations	Understand the operation of division as sharing equally and grouping Begin to relate division and fractions	Understand the operation of division as sharing and grouping Relate division and fractions Understand the idea of a remainder and make sensible decisions about rounding up or down after division in the context of a problem	continue to understand the operation of division as sharing and grouping Relate division and fractions begin to understand ratio problems continue to make sensible decisions about rounding up or down after division in the context of a problem	continue to relate division and fractions Understand • scaling by simple fractions • simple rates • begin to understand ratio problems interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding	continue to relate division and fractions Continue to understand • scaling by fractions • rate • ratio problems interpret remainders as whole number remainders, fractions, decimals or by rounding, as appropriate for the context round answers to a specified degree of accuracy
		show that division of one number by another cannot be done in any order	understand that the principles of the commutative and associative laws do not apply to division	understand the distributive law continue to understand that the principles of the commutative and associative laws do not apply to division	continue to understand the distributive law	**use their knowledge of the order of operations**
	.	recognise the inverse relationship between multiplication and division	understand the inverse relationship between multiplication and division	continue to understand the inverse relationship between multiplication and division		
record using marks that they can interpret and explain	use pictorial representations	write mathematical statements using the division (÷), and equals (=) signs	solve missing numbers problems involving division	continue to solve missing number problems	continue to solve missing number problems begin to use brackets	continue to solve missing number problems explore the order of operations using brackets

begin to use the vocabulary involved in dividing *share, halve*	begin to use the vocabulary involved in dividing *array, row, column, equal groups of,*	understand and use the vocabulary involved in dividing *divide, left over*	understand, read and spell vocabulary related to division correctly *in every, remainder*	understand, read and spell vocabulary related to division correctly *for every, quotient, divisible by, factor*	read, spell and pronounce mathematical vocabulary related to division correctly *prime numbers, prime factors, composite numbers,* *common factors*	read, spell and pronounce mathematical vocabulary related to division correctly *common multiple*

Recalling number facts						
begin to count in twos and tens	**count in multiples of twos, fives and tens**	**count in steps of 2, 3, and 5 from 0**	**count from 0 in multiples of 4, 8, 50 and 100**	**count in multiples of 6, 7, 9, 25 and 1000**	use knowledge of counting in multiples to count in decimal steps (one decimal place)	use knowledge of counting in multiples to count in decimal steps (two decimal places)
know corresponding halves of doubles of all numbers to 5	know corresponding halves of doubles of all numbers to 10	recall corresponding halves of doubles of all numbers to 15 and doubles of multiples of 5 to 50	recall corresponding halves of doubles of all numbers to 20, doubles of multiples of 5 to 100 and doubles of multiples of 100 to 500	derive corresponding halves of doubles of multiples of 50 to 1000 and multiples of 1000	derive corresponding halves of doubles of decimals (to one decimal place) using knowledge of place value	derive corresponding halves of doubles of decimals (to two decimal places) using knowledge of place value
	begin to recognise odd and even numbers	**recall and use division facts for the 2, 5 and 10 multiplication tables recognise odd and even numbers**	**recall and use division facts for the 3, 4, 8 multiplication tables** and begin to use knowledge of place value to derive related facts	**recall division facts for multiplication tables up to 12 × 12, and use place value to derive related facts**	continue to recall division facts for multiplication tables up to 12 × 12 fluently, and derive and use related facts	continue to recall division facts for multiplication tables up to 12 × 12 fluently, and derive and use related facts
				recognise and use factor pairs	**identify multiples and factors, and common factors of two numbers, and primes**	**identify common factors, common multiples and prime numbers**
Mental methods and mental methods with jottings						
count a set of objects by grouping in 2s solve simple problems involving halving and sharing	count a set of objects by grouping in 2s, 5s or 10s solve problems involving sharing, grouping and halving; make equal groups	**calculate mathematical statements for division within the multiplication tables**	**calculate mathematical statements for division using the multiplication tables that they know, beginning to divide two-digit numbers by one-digit numbers (for known multiplication tables)**	**divide mentally using place value, known and derived facts, including dividing by 1**	**divide numbers mentally drawing upon known facts** use factors to construct equivalence statements begin to divide tenths, and 1-digit whole numbers and tenths by 1-digit whole numbers	**perform mental calculations, including with mixed operations, large numbers and decimals**
Formal written layout						
				begin to divide two-digit and three-digit numbers by a one-digit number using formal written layout	**divide numbers up to 4 digits by a one-digit number using a formal written method of short division and interpret remainders appropriately for the context**	**divide numbers up to 4 digits by a two-digit whole number using a formal written method**

						divide numbers (up to two decimal places) by 1-digit and 2-digit whole numbers give answers up to 2 decimal places calculate decimal fraction equivalents
Estimating and checking						
			estimate the answer to a calculation use inverse operations to check answers use equivalent calculations to check answers	estimate the answer to a calculation use inverse operations to check answers use equivalent calculations to check answers	use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy continue to use appropriate strategies to check answers	use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy. continue to use appropriate strategies to check answers

DIVISION: Y1

Understanding the operation and related vocabulary.	Mental Calculations	Written Calculations
<u>**Understanding the operation**</u> Begin to understand division as both sharing and grouping using concrete objects, pictorial representations and arrays to solve problems. Children should begin to explore finding simple fractions of objects, numbers and quantities. <u>**Vocabulary**</u> Begin to use the vocabulary involved in dividing: share, share equally, one each, two each…, group, groups of, lots of, array, row, column, equal groups of <u>**Generalisations**</u> • True or false? I can only halve even numbers. • Grouping and sharing are different types of problems. Some problems need solving by grouping and some by sharing. Encourage children to practically work out which they are doing. <u>**Some Key Questions**</u> How many groups of…? How many in each group? Share… equally into… What can do you notice?	<u>**Number facts**</u> Experience <u>regular counting</u> on and back from different numbers in 1s and in multiples of 2, 5 and 10. Count a set of objects by grouping in 2s, 5s or 10s *Count these pennies (2 at a time)* Know corresponding halves of doubles of all numbers to 10: Half of 6 is ☐ Half of 10 is ☐ Begin to recognise odd and even numbers. Use cubes to make 9 and recognise it is odd (as the cubes cannot be paired)  They should begin to recognise the number of groups counted to support understanding of relationship between multiplication and division.  	No formal written layout. Children record their maths using pictorial representations, arrays, number lines and mathematical statements. 10 ÷ 5 = 2   Use of arrays as a pictorial representation for division. 15 ÷ 3 = 5 There are 5 groups of 3. 15 ÷ 5 = 3 There are 3 groups of 5. 

Mental methods and jottings

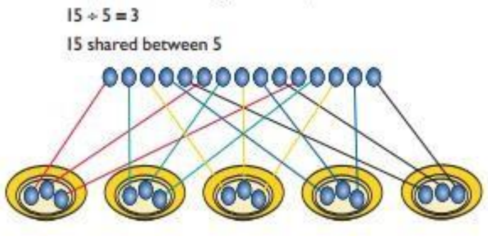
Solve problems involving sharing, grouping and halving; make equal groups

Counting on

There are 10 seeds and some flower pots. Each pot needs 2 seeds in it. How many pots can be planted?

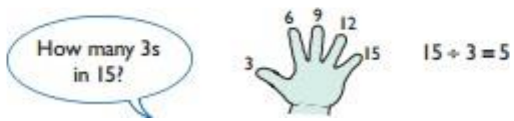
Sharing

Develops importance of one-to-one correspondence.



Grouping

Children should apply their counting skills to develop some understanding of grouping.



How many groups of 2 are in 6?

Jo has 12 Lego wheels. How many cars can she make?

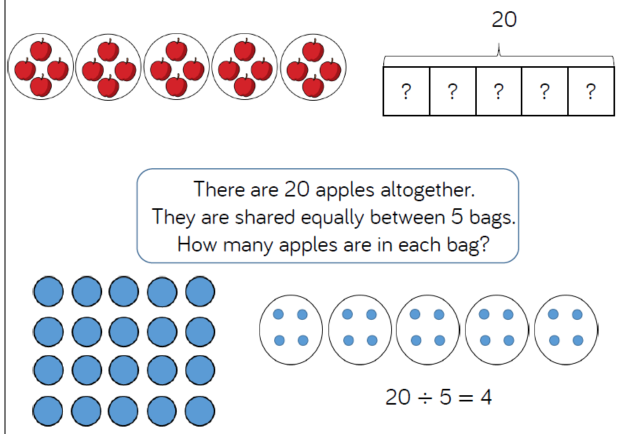
Using doubling and halving

Know corresponding halves of doubles to 10.

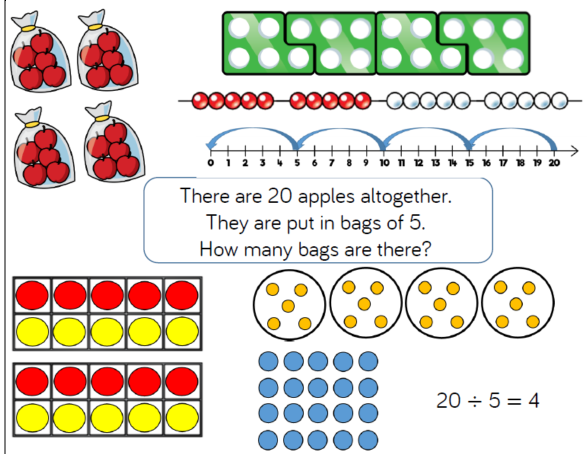
Half of 10 is 5.

A ladybird has 12 spots altogether. How many spots on each side of its body?

Sharing (also applicable in year 2)



Grouping (also applicable in year 2)



<table>
<tr><th colspan="3">DIVISION: Y2</th></tr>
<tr><th>Understanding the operation and related vocabulary.</th><th>Mental Calculations</th><th>Written Calculations</th></tr>
<tr>
<td>

<u>Understanding the operation</u>
Continue to understand division as both sharing and grouping using concrete objects, pictorial representations and arrays to solve problems.

Begin to relate division to fractions.

Continue to work on arrays and begin to understand the inverse relationship between x and ÷.
15 ÷ 3 = 5 There are 5 groups of 3.
5 x 3 = 15
15 ÷ 5 = 3 There are 3 groups of 5.
3 x 5 = 15

Show that division of one number by another cannot be done in any order.
15 ÷ 5 = 3
5 ÷ 15 ≠ 3

Write mathematical statements using the division and equals sign.
6 ÷ 2 = □   □ = 6 ÷ 2
6 ÷ □ = 3   3 = 6 ÷ □
□ ÷ 2 = 3   3 = □ ÷ 2
□ ÷ ∇ = 3   3 = □ ÷ ∇

<u>Vocabulary</u>
Understand and use the vocabulary related to division:

Group in pairs, 3s … 10s etc
equal groups of, divide, ÷, divided by, divided into, remainder, left over.
<u>Generalisations</u>

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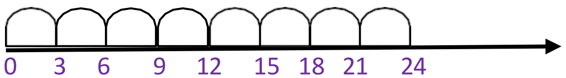
<u>Number facts</u>
Count regularly, on and back, in steps of 2, 3, 5 and 10 from 0.
0   3 6 9 12 15 18 ……30
50 45 40 35 30 …….. 0

Recall and use division facts for the 2, 5 and 10 times table:
How many groups of 10 in 30?
Divide 14 by 2.
25 divided by 5.

Recall corresponding halve of doubles of all numbers to 15 and doubles of multiples of 5 to 50.
Half of 14 is □
Half of 30 is □

Recall and use division facts for the 2, 5 and 10 times table.
How many groups of ten in 30   divide14 by 2   25 divided by 5

Recognize odd and even numbers.
Explain why 15 is an odd number

<u>Mental methods and jottings</u>
<u>Counting on</u>
70 ÷ 10 = 7 (by counting on in tens using fingers to keep track).
<u>With jottings:</u>
24 ÷ 3 = 8 (counting on in threes using a number line to keep track).


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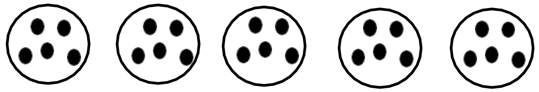
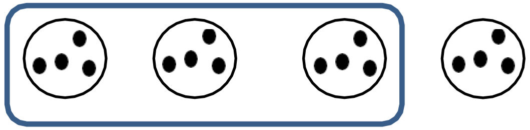
No formal written layout.

Children record their maths using pictorial representations, number lines and mathematical statements.

<u>Use knowledge of times table facts to recall inverse division</u>
4 x 10 = 40
40 ÷ 10 = 4

<u>Sharing (further examples)</u>
25 ÷ 5 = 5 → share 25 between 5 groups

Apply this to fractions:
3/4 of 16 = 12→ share 16 between 4 groups, count how many are in 3 groups.


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Noticing how counting in multiples if 2, 5 and 10 relates to the number of groups you have counted (introducing times tables)

An understanding of the more you share between, the less each person will get (e.g. would you prefer to share these grapes between 2 people or 3 people? Why?)

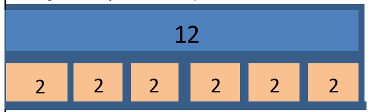
Secure understanding of grouping means you count the number of groups you have made. Whereas sharing means you count the number of objects in each group.

Some Key Questions

How many 10s can you subtract from 60?
I think of a number and double it. My answer is 8. What was my number?
If 12 x 2 = 24, what is 24 ÷ 2?
Questions in the context of money and measures (e.g. how many 10p coins do I need to have 60p? How many 100ml cups will I need to reach 600ml?)

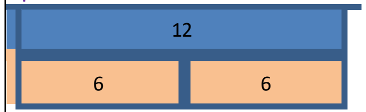
Sharing (further examples)

Share 12 pencils **equally** between 6 pots (using objects/pictures)



Grouping (further examples)

12 pencils shared between 2 po ts, how many in each pot?



Using doubling and halving

Know corresponding halves of doubles of all numbers to 15 and doubles of all numbers of multiples of 5 to 50.

14 ÷ 2 = 7 (by recalling the doubles first)

With Jottings

24÷2 (by halving 20, halving 4 and recombining)

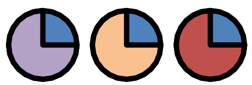
Using known facts and place value

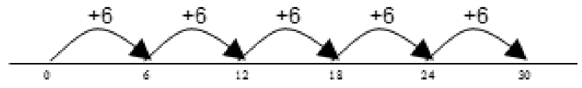
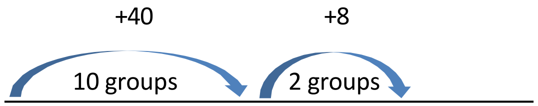
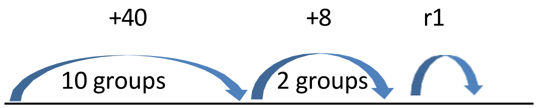
If 4 ÷ 2 = 2 then 40 ÷ 2 = 20

Fractions

Find a half, a quarter and a third of shapes, objects, numbers and quantities. Finding a fraction of a number of objects to be related to sharing.
Explore visually and understand how some fractions are equivalent – e.g. two quarters is the same as one half.

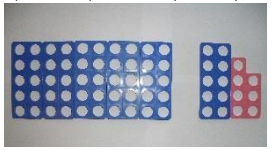
3 apples shared between 4 people = $\frac{3}{4}$



DIVISION: Y3		
Understanding the operation and related vocabulary.	**Mental Calculations**	**Written Calculations**
<u>Understanding the operation</u> Understand the operation of division as sharing and grouping. Understand the principles of commutative and associative laws **do not** apply to division. Recognise that 24÷4 is not equal to 4÷24 Understand the inverse relationship between multiplication and division. 6 x 3 = 18 3 x 6 = 18 18 = 3 x 6 18 = 6 x 3 18 ÷ 3 = 6 18 ÷ 6 = 3 6 = 18 ÷ 3 3 = 18 ÷ 6 Continue using a range of missing number equations as in year 2 but with appropriate numbers. 15 ÷ □ = 5 □ = 14 ÷ 2 20 = □ x □ 5 + 10 = 35 ÷ □ 7 < □ ÷ 2 □ ÷ □ > 8 Continue to relate fractions to division. ¼ of 16 = 16 ÷ 4 **<u>Vocabulary</u>** Inverse, in every **<u>Generalisations</u>** Inverses and related facts – develop fluency in finding related multiplication and division facts. Develop the knowledge that the inverse relationship can be used as a checking method. **<u>Some Key Questions</u>** Questions in the context of money and measures that involve remainders (e.g. How many lengths of 10cm	**<u>Number facts</u>** Count regularly, on and back, in steps of 3, 4 and 8. Count from 0 in multiples of 4, 8, 50 and 100. 0 8 16 24 32 …. 500 450 400 350 …. Recall and use division facts for the 3, 4 and 8 times table. How many threes in 27? Divide 24 by 4 48 divided by 8 Divide 80 in to fours Recall corresponding halves and doubles of all numbers to 20, doubles of multiples of 5 to 100 and doubles of multiples of 100 to 500. Half of 16 is ☐ 18÷2=☐ Half of 70 is ☐ **<u>Mental methods and jottings</u>** Calculate mathematical statements for division using the multiplication tables that they know, beginning to divide two-digit numbers by one-digit numbers (for known multiplication tables). **<u>Counting on</u>** 70 ÷ 5 (by counting on in fives from 50) With jottings: 52 ÷ 4 (by counting on in fours from 4 x 10 using a number line to keep track). With remainders: 54 ÷ 4 **<u>Partitioning</u>**	No formal written layout. Begin to divide 2 digit numbers by one digit numbers (for known multiplication tables). **<u>Grouping</u>** How many 6's are in 30? 30 ÷ 6 can be modelled as:  **<u>Becoming more efficient using a number line</u>** Children need to be able to partition the dividend in different ways. **48 ÷ 4 = 12**  **<u>Remainders</u>** **49 ÷ 4 = 12 r1**  Make sensible decisions about rounding up or down after division in the context of a problem. **Sharing:** 49 shared between 4. How many left over? **Grouping:** How many 4s make 49. How many are left over?

can I cut from 81cm of string? You have £54. How many £10 teddies can you buy?)
What is the missing number?
17 = 5 x 3 + __
___= 2 x 8 + 1

Without crossing the tens boundary:
69 ÷ 3 = 23
(60 ÷ 3 = 20 ; 9 ÷ 3 = 3)
20 + 3 = 23
Partition number in different ways:
52 = 50 + 2; 40 + 12; 30 + 12 etc
With jottings
Partitioning crossing the tens boundary.
65 ÷ 5 = 13
(12 x 5)   (1 x 5)



With remainders: 67 ÷ 5 = 13r2

Doubling and halving
84 ÷ 2 = 42 *(80 ÷ 2 = 40) (4 ÷ 2 = 2)*
With jottings
100 ÷ 4 = 25 (halve and halve again)
Half of 100 is 50, half of 50 is 25.

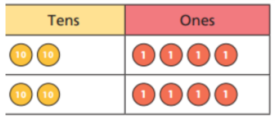
Known facts and place value
Use multiplication and division facts they know to make links with other facts.
If: 3 x 2 = 6, 6 ÷ 3 = 2, 2 = 6 ÷ 3
Then: 30 x 2 = 60, 60 ÷ 3 = 20, 2 = 60 ÷ 30

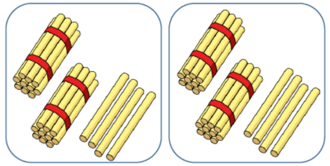
Estimating
Estimate the answer to a calculation:
52 ÷ 4 is between 10 fours and 20 fours.

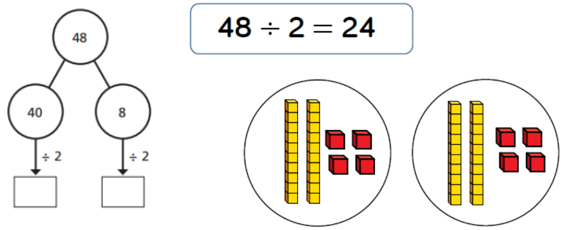
Use inverse operations and equivalent calculations to check answers:
Check 65 ÷ 5 = 13 with 5 x 13 = 65.

Place value counters can be used to support children apply their knowledge of grouping.
60 ÷ 10 = How many groups of 10 in 60?
600 ÷ 100 = How many groups of 100 in 600?
Remainders
Understand the idea of a remainder and make sensible decisions about rounding up or down after division in the context of a problem.

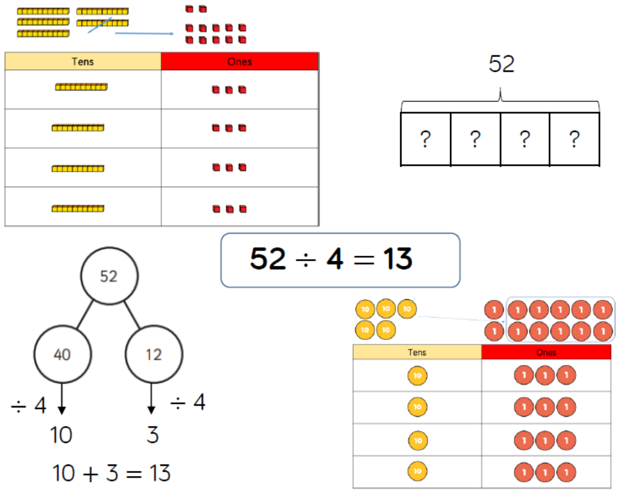
Divide 2-digit by 1-digit (sharing with no exchange or remainders)



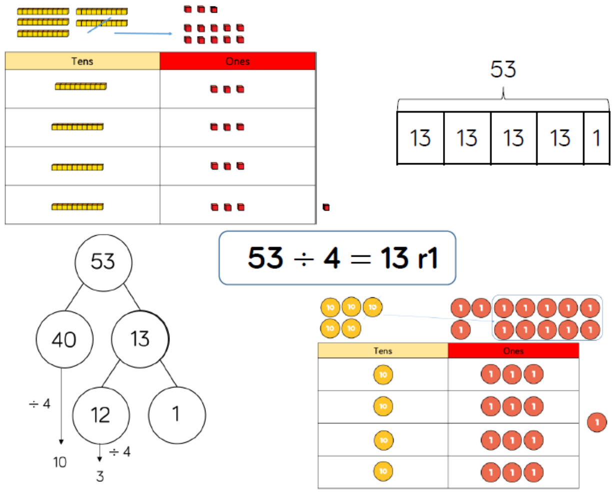




Divide 2-digit by 1-digit (sharing with exchange but no remainders – also applicable in year 4)



<u>Divide 2-digit by 1-digit (sharing with exchange and remainders – also applicable in year 4)</u>



DIVISION: Y4

Understanding the operation and related vocabulary.

Understanding the operation

Continue to understand the operation of division as sharing and grouping.

Relate division and fractions.

1/3 = 1 ÷ 3   2/3 = 2 ÷ 3

Understand links to ratio problems (2 quantities in a fixed ratio.

Continue to understand the principles of commutative and associative laws **do not** apply to division.

Understand the distributive law and recognise that

65 ÷ 5 is the same as (50 ÷ 5) + (15 ÷ 5)

Continue to understand the inverse relationship between multiplication and division.

6 x 7 = 42   7 x 6 = 42   42 = 7 x 6   42 = 6 x 7

42 ÷ 7 = 6   42 ÷ 6 = 7   7 = 42 ÷ 6   6 = 42 ÷ 7

Continue using a range of equations as in year 3 but with appropriate numbers.

54÷☐= 6   ☐=80x8   48=☐x☐

36÷4 = 18÷☐   5 < ☐ ÷9   ☐÷☐ > 11

Vocabulary

divide, divided by, divisible by, divided into
share between, groups of, factor, factor pair, multiple
times as (big, long, wide …etc), for every, quotient
equals, remainder, quotient, divisor
inverse

Generalisations

Mental Calculations

Number facts

Count on and back in multiples of 6, 7, 9, 25 and 1000.

0 7 14 21 28 …

300 275 250 225 200 …

Learn the multiplication facts to 12 x 12 and use place value to derive related facts.

6 x 7 = 42   70 x 6 = 420

42 ÷ 6 = 7   420 ÷ 6 = 70

How many sixes in 54?

Divide 63 by 7

350 divided by 5

108 ÷ 12, what is the quotient?

Recognise and use factor pairs

List the factor pairs of 32

Derive corresponding halves of doubles of multiples of 50 to 1000 and multiples of 1000.

Half of 150 is ☐   700÷2=☐   6000÷2=☐

Mental methods and jottings

Divide mentally using place value, known and derived facts including dividing by 1.

Counting on

126 ÷ 6 (by counting on in sixes from 120).

With jottings

161 ÷ 7 (by counting on in sevens from 7 x 20 using a number line to keep track)

With remainders: 163 ÷ 7

Partitioning

Without crossing the tens boundary:

Written Calculations

Begin to divide 2-digit and 3-digit numbers by a 1-digit number using a formal written layout.

e.g.

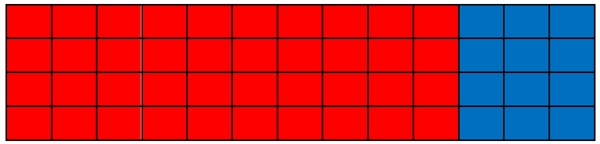
98 ÷ 7

138 ÷ 3

Towards a formal written method

Alongside pictorial representations and the use of models and images, children should progress onto short division using a bus stop method.

52 ÷ 4



4	40	12

	10	**3**	**= 13**
4	40	12	

True or false? Dividing by 10 is the same as dividing by 2 and then dividing by 5. Can you find any more rules like this?

Is it sometimes, always or never true that □ ÷ ∆ = ∆ ÷ □?

Inverses and deriving facts. ‘Know one, get lots free!’ e.g.: 2 x 3 = 6, so 3 x 2 = 6, 6 ÷ 2 = 3, 60 ÷ 20 = 3, 600 ÷ 3 = 200 etc.

Sometimes, always, never true questions about multiples and divisibility. (When looking at the examples on this page, remember that they **may not** be ‘always true’!) E.g.:

- Multiples of 5 end in 0 or 5.
- The digital root of a multiple of 3 will be 3, 6 or 9.
- The sum of 4 even numbers is divisible by 4.

78 ÷ 6 = 13 Partition in to multiples of the divisor

60 ÷ 6 = 10 **;** 18 ÷ 6 = 3

10 + 3 = 13

Using Numicon, dienes or place value counters as support.

With jottings

Partitioning crossing the tens boundary.

185 ÷ 5 = 37   (150 ÷ 5 = 30; 35 ÷ 5 = 7)

30 + 7 = 37

With remainders: 187 ÷ 5

Continue to partition number in different ways:

762 = 700 + 60 + 2; 600 + 120 + 42

Doubling and halving

600÷4 (halve & halve again)

Half of 600 is 300, half of 300 is 150

With jottings

112 ÷ 8 (halve, halve and halve again)

Half of 112 = 56, half of 56 = 28, half of 28 = 14

Factors

500 ÷ 20 (Divide 500 by 10 then divide by 2)

With jottings

90 ÷ 6 (Divide 90 by 3 then divide by 2)

Estimating

Estimate the answer to a calculation:

138 ÷ 3 is between 40 threes and 50 threes.

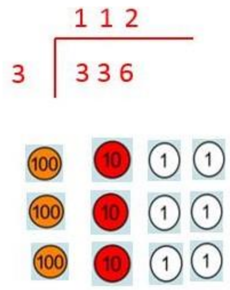
Use inverse operations and equivalent calculations to check answers:

Check 98 ÷7 = 14 with 7 x 14 = 98

Short division can also be modelled for understanding using place value counters as shown below.

Calculations with 2 and 3-digit dividends. E.g.

336 ÷ 3



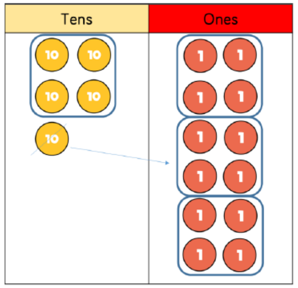
$$\begin{array}{r|l} & 1\quad 2\quad 7 \\ \hline 6 & 7\ {}^{1}6\ {}^{4}2 \end{array}$$

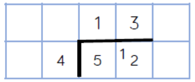
Remainders

Continue to make sensible decisions about rounding up or down after division in the context of a problem.

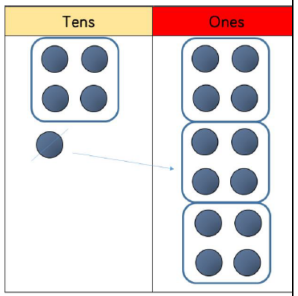
Pictorial leading to written

Divide 2-digits by 1-digit (grouping)



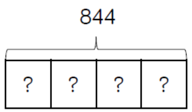


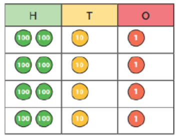
$52 \div 4 = 13$

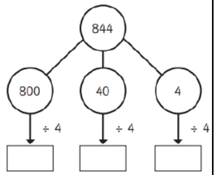


Divide 3-digits by 1-digit (sharing)

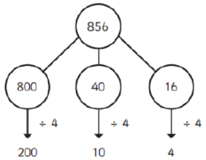
$844 \div 4 = 211$

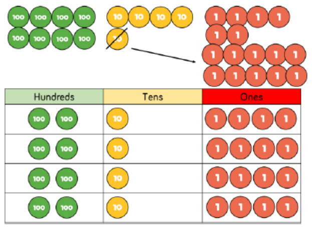




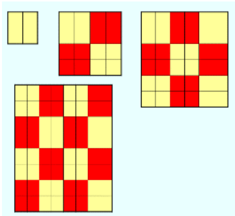


$856 \div 4 = 214$



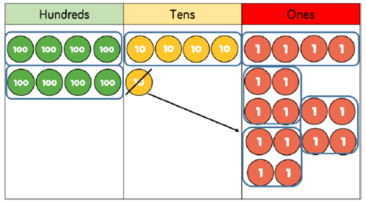


DIVISION: Y5		
Understanding the operation and related vocabulary.	**Mental Calculations**	**Written Calculations**
Understanding the operation Continue to understand the distributive law and recognise that 65 ÷ 5 is the same as (50 ÷ 5) + (15 ÷ 5) Continue to relate fractions and division. Understand: - Scaling by simple fractions - Simple rates - Begin to understand links to ratio problems. - Continue using a range of equations as in year 4 but with appropriate numbers. □= 540 ÷6 □=3.2÷8 48=□÷□ 90÷30 = 6x□ □x□ > 600÷8 Continue to solve missing number problems □= 540 ÷6 □=3.2÷8 48=□÷□ 90÷30 = 6x□ □x□ > 600÷8 Begin to use brackets. (60+3) ÷ 7 = □ □=10 + (1.4 ÷2) **Vocabulary** common factors prime number, prime factors composite numbers short division square number cube number inverse power of **Generalisations**	**Number facts** Count regularly using a range of multiples, and powers of 10, 100 and 1000, building fluency. Practice and apply the multiplication facts to 12 x 12. Use knowledge of counting in multiples to counting in decimal steps (one decimal place). 0.6 1.2 1.8 2.4 …. Derive corresponding halves of doubles of decimals (to 1 place) using knowledge of place value. Half of 0.4 = 0.2 3.6 ÷ 2 = 1.8 Continue to recall division facts for multiplication tables to 12 x 12 fluently and derive and use related facts: 560 divided by 7 divide 2.1 by 7 4500 ÷ 5, what is the quotient? 3.2 divided by 4 Identify multiples and factors and common factors of two numbers and primes. list the multiples of 9 between 150 and 180 (using tests of divisibility) **Mental methods and jottings** Divide mentally drawing upon known number facts. Use factors to construct equivalence statements. Begin to divide tenths and 1-digit whole numbers and tenths by 1-digit whole numbers. **Partitioning** Using distributive law: 546 ÷ 6 (540 ÷ 6 = 90; 6 ÷ 6 = 1 so 90 + 1 = 91) With jottings	Divide numbers up to 4 -digits by a 1-digit number using a formal written method (short division) and interpret remainders appropriately for the context e.g. 3075 ÷ 5; 6831 ÷ 9 0 6 1 1 r 2 5 | 3 0 5 7 **Remainders** Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding. (See NCETM video – Division with exchange) **See year 6 for further examples of short division**

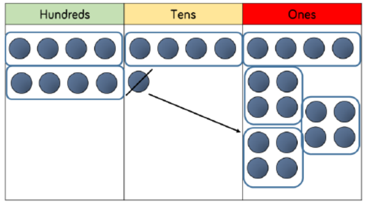
The = sign means equality. Take it in turn to change one side of this equation, using multiplication and division, e.g. Start: **24 = 24** Player 1: **4 x 6 = 24** Player 2: **4 x 6 = 12 x 2** Player 1: **48 ÷ 2 = 12 x 2** Sometimes, always, never true questions about multiples and divisibility. E.g.: • If the last two digits of a number are divisible by 4, the number will be divisible by 4. • If the digital root of a number is 9, the number will be divisible by 9. • When you square an even number the result will be divisible by 4 (one example of ‘proof’ shown left) 	24.5 ÷ 7 (21 ÷ 7 = 3; 3.5 ÷ 7 = 0.5 so 3 + 0.5 = 3.5) Continue to partition number in different ways: 762 = 700 + 60 + 2; 600 + 120 + 42 **Doubling and halving** 14.8 ÷ 4 (halve and halve again) Half of 14.8 = 7.4; half of 7.4 = 3.7 *With jottings*: 3800 ÷ 50 (divide by 100 then double) 3800 ÷ 100 = 38; double 38 = 76. **Factors** 84÷20 (halve and divide by 10) 84÷2=42 42÷10=4.2 With jottings 150 ÷ 6 (150 ÷ 3 = 50, then 50 ÷ 2 = 25). **Using known facts and place value** 8.4 ÷ 7 (multiply dividend by 10, then divide quotient by 10) 84÷7 =12, 12÷10=1.2 **Estimating** Use rounding to check answers to calculation and determine, in the context of a problem, levels of accuracy: 256 ÷ 12 is approximately 2560 ÷ 10. Continue to use appropriate strategies to check answers: Check 860 ÷ 9 by using the inverse.	

Pictorial leading to written

Divide 3-digits by 1-digit (grouping)

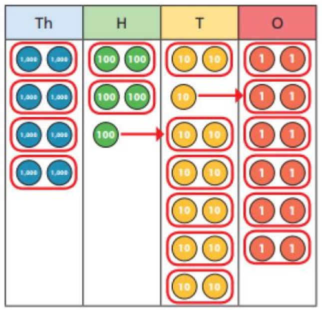


		2	1	4
	4	8	5	$^{1}6$



856 ÷ 4 = 214

Divide 4-digits by 1-digit (grouping)



	4	2	6	6
2	8	5	$^{1}3$	$^{1}2$

8,532 ÷ 2 = 4,266

DIVISION: Y6

Understanding the operation and related vocabulary.

Understanding the operation
Continue to relate fractions and division.

Understand:

- Scaling by simple fractions
- Simple rates
- Begin to understand links to ratio problems.

Use their knowledge of order of operations.

Understand that when there are no brackets, do multiplication or division before addition or subtraction.

Understand that if the examples are at the same level of priority then work out the examples from left to right.

Continue using a range of equations as in year 5 but with appropriate numbers.
☐= 540 ÷0.6   ☐=0.48÷8   4.8=☐÷☐
9÷0.3 = 6x☐   ☐x☐ > 0.56÷8

Explore the order of operations using brackets.
compare 14 ÷ (2 x 5) with (14 ÷ 2) x 5

Vocabulary
Common multiple

Generalisations
Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an

Mental Calculations

Number facts
Children should count regularly, building on previous work in previous years.

Use knowledge of counting in multiples to counting in decimal steps (two decimal places).
0.09 0.18 0.27 0.36 …

Continue to recall division facts for multiplication tables to 12 x 12 fluently and derive and use related facts:
3000 divided by 60   divide 0.12 by 6
5800 ÷ 6, what is the quotient?
0.64 divided by 8

Derive corresponding halves of decimals (to 2 places) using knowledge of place value.
Half of 0.48 is ☐   0.74÷2=☐

Using known facts and place value:
0.99 ÷ 11 (multiply dividend by 100, then divide quotient by 100)
99 ÷ 11 = 9, 9 ÷ 100 = 0.09

Identify common factors, common multiples and prime numbers.
15 ÷ 6 (divide by 3 then 2)
15 ÷ 3 = 5 5 ÷ 2 = 2.5

Mental methods and jottings
Perform mental calculations, including with mixed operations, large numbers and decimals.

Partitioning
Using distributive law:

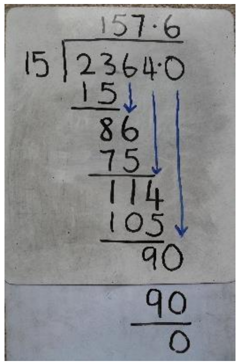
Written Calculations

Divide numbers up to 4 digits by a 1 digit or 2-digit whole number using a formal written method (short division and long division).

Divide numbers up to two decimal places by 1-digit and 2-digit whole numbers.
Give answers up to 2 decimal places.
Calculate decimal fractions e.g.

Short division:
56.4 ÷ 4; 5246 ÷ 22; 19.88 ÷ 7; 1504 ÷ 8

Long division:
2360 ÷ 15; 187.5 ÷ 15



acrostic such as PEMDAS, or could be encouraged to design their own ways of remembering.

Sometimes, always, never true questions about multiples and divisibility. E.g.: If a number is divisible by 3 and 4, it will also be divisible by 12. (also see year 4 and 5, and the hyperlink from the Y5 column)

Using what you know about rules of divisibility, do you think 7919 is a prime number? Explain your answer.

18.12 ÷ 3 (18 ÷ 3 = 6; 0.12 ÷ 3 = 0.4 so 6 + 0.4 = 6.4)
With jottings
2.58 ÷ 6 (2.4 ÷ 6 = 0.4; 0.18 ÷ 6 = 0.03 so 0.4 + 0.03 = 0.43)

Doubling and halving
9.6 ÷ 40 (halve and halve again and divide by 10)
Half of 9.6 = 4.8; half of 4.8 = 2.4; 2.4 ÷ 10 = 0.24
With jottings:
1700 ÷ 25 (divide by 100 then double and double)
1700 ÷ 100 = 17; double 17 = 34; double 34 is 68

Using known facts and place value
0.99÷11 (multiply dividend by 100, then divide quotient by 100)
99÷11=9, 9÷100=0.09

Factors
15 ÷ 6 (divide by 3 then 2)
15 ÷ 3 = 5   5 ÷ 2 = 2.5
With jottings
900 ÷ 12 (900 ÷ 3 = 300, then 300 ÷ 2 = 150 then 150 ÷ 2 = 75)

Estimating
Use estimation to check answers to calculation and determine, in the context of a problem, levels of accuracy:
5872 ÷ 54 is approximately 6000 ÷ 50.

Continue to use appropriate strategies to check answers:
Check 4581 ÷ 27 by using the inverse.

Remainders
Interpret remainders as whole number remainders, fractions, decimals or by rounding, as appropriate for the context.

Further examples

Divide multi digits by 2-digits (short division)

		0	3	6
	12	4	$^{4}3$	$^{7}2$

432 ÷ 12 = 36

7,335 ÷ 15 = 489

	0	4	8	9
15	7	$^{7}3$	$^{13}3$	$^{13}5$

15	30	45	60	75	90	105	120	135	150

Divide multi digits by 2-digits (long division)

		0	3	6	
1	2	4	3	2	(×30)
	−	3	6	0	
			7	2	(×6)
	−		7	2	
				0	

12 × 1 = 12
12 × 2 = 24
12 × 3 = 36
12 × 4 = 48
12 × 5 = 60
12 × 6 = 72
12 × 7 = 84
12 × 8 = 96
12 × 7 = 108
12 × 10 = 120

432 ÷ 12 = 36

7,335 ÷ 15 = 489

	0	4	8	9	
15	7	3	3	5	
−	6	0	0	0	(×400
	1	3	3	5	
−	1	2	0	0	(×80)
		1	3	5	
−		1	3	5	(×9)
				0	

1 × 15 = 15
2 × 15 = 30
3 × 15 = 45
4 × 15 = 60
5 × 15 = 75
10 × 15 = 150

OR

$372 \div 15 = 24 \text{ r}12$

			2	4	r	1	2
1	5	3	7	2			
	−	3	0	0			
			7	2			
	−		6	0			
			1	2			

$1 \times 15 = 15$
$2 \times 15 = 30$
$3 \times 15 = 45$
$4 \times 15 = 60$
$5 \times 15 = 75$
$10 \times 15 = 150$

			2	4	$\frac{4}{5}$
1	5	3	7	2	
	−	3	0	0	
			7	2	
	−		6	0	
			1	2	

$372 \div 15 = 24\frac{4}{5}$